

Биноми, змешане и грешно пред; Купуват Купува мат. анализ
 Бу Багун, Буи Ст Задачи по мат. анализ
 Купу-примену в анализе

Ландо,
 Метод. раз-ка
 по мат. анализ
 вилковски точки.

О: $f(x) = O(g(x))$, ели $x \in D$ $|f(x)| \leq M|g(x)|$;

$f(x) = O(1)$ на D - оп. груп-а

$f(x) = O(g(x))$ в т.х, ели $\exists \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = 0$.

$f(x) = O(g(x))$, $\exists \alpha(x)$; $f(x) = \alpha(x)g(x)$, $\lim_{x \rightarrow x_0} \alpha(x) = 0$

$f \sim g \Leftrightarrow \exists \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = 1 \Leftrightarrow f(x) = g(x) + O(g(x))$; ; $\frac{f(x) - g(x)}{g(x)} \rightarrow 0$
 знач. $x \rightarrow x_0$
 эквивалент.
 раз-ли

$\frac{f(x)}{g(x)} - 1 = \alpha(x) \rightarrow 0 \Rightarrow f(x) = g(x) + O(g(x))$

$\Leftrightarrow f(x) - g(x) = O(g(x))$,

$\frac{f(x)}{g(x)} - 1 = \frac{O(g(x))}{g(x)} = O(1)$.

Множ. сравнения: $f_n(x) = (x-x_0)^n$; $f_n = O(f_{n-1})$; $\frac{f_n}{f_{n-1}} = (x-x_0) \rightarrow 0$
 Степенные функции

Множ. разл.: $D = \{x > 1\}$;

1) $\ln^{\alpha} x$, $\alpha_1 < \alpha_2$ $\ln^{\alpha_1} < \ln^{\alpha_2}$ логарифм.

2) x^{α} степен.

3) a^x , $a > 1$ экспонен.

Ели $f(x)$ прот. $g(x)$ в списке, то $\frac{f(x)}{g(x)} \xrightarrow{x \rightarrow \infty} 0$

К. Коши $\sum a_k$ сход-е $\Leftrightarrow$ гл. частичные сумм.

2549

$$S_n = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)} = 1 - \frac{1}{n+1} \quad \lim_{n \rightarrow \infty} S_n = 1$$

2550

$$S = \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{(3n-2)(3n+1)} \quad ; \quad \frac{1}{(3n-2)(3n+1)} = \frac{A}{3n-2} + \frac{B}{3n+1}$$

$$1 = 3n \cdot A + A + 3n \cdot B - 2B \quad ; \quad \begin{cases} A = \frac{1}{3} \\ B = -\frac{1}{3} \end{cases} \quad S_n = \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{(3n-2)(3n+1)} \Rightarrow$$

$$\lim_{n \rightarrow \infty} S_n = \frac{1}{3}$$

2559

$$\sum_{n=1}^{\infty} \frac{1}{2n-1} > \sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n};$$

Δ2588

$$\sum_{n=1}^{\infty} \frac{1}{100n+1}, 100n+1 < 100(n+1) \quad \Rightarrow \quad \sum_{n=1}^{\infty} \frac{1}{100n+1} > \frac{1}{100} \sum_{n=1}^{\infty} \frac{1}{n+1}$$

Δ2564

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{(2n-1)(2n+1)}} > \sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}$$

$$\sqrt{(2n-1)(2n+1)} < \frac{2n-1+2n+1}{2} = 2n$$

Δ2570

$$\sum_{n=1}^{\infty} a_n; \quad \exists \lim_{n \rightarrow \infty} n a_n = b$$

$\delta = \epsilon/2$, то по теореме

$$\forall \epsilon > 0 \quad \exists N(\epsilon) > 0 \quad \forall n > N_0 \quad |n a_n - b| < \epsilon$$

$$-\epsilon < n a_n - b < \epsilon; \quad n a_n > b - \epsilon > b/2$$

$$a_n > \frac{b}{2n}$$

$$\sum_{n=1}^{\infty} a_n > \frac{b}{2} \sum_{n=1}^{\infty} \frac{1}{n} - \text{расход.}$$

Δ2549

$$\text{Если } \sum_{n=1}^{\infty} a_n \text{ расход., } a_n > 0, \lim_{n \rightarrow \infty} n a_n = 0, \quad n a_n = o(1);$$

$$\rho a_{n+p} + \dots + a_{n+p} < \epsilon; \quad \text{то } n a_{2n} < \epsilon$$

$$(n+1) a_{2n+1} < \epsilon.$$

пр. Даламбера

$$\sum_{n=1}^{\infty} u_n, \text{ если } \exists \lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = q \text{ и } 0 < q < 1, \text{ то расход.-с.}$$

$$\text{Если } \exists \lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = q_1, 0 < q_1 < 1; \text{ если } \exists \lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} > 1, \text{ то расход.-с.}$$

пр. Коши

$$\lim_{n \rightarrow \infty} \sqrt[n]{u_n} = q \text{ и } q < 1, \text{ то расход.-с.}$$

сум. пр. Коши

$$\rightarrow \lim_{n \rightarrow \infty} \dots$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n!}, \quad u_n = \frac{x^n}{n!}; \quad \lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = \lim_{n \rightarrow \infty} \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} = \lim_{n \rightarrow \infty} \frac{x}{n+1} = 0, \text{ то расход.-с.}$$

$$\sum_{n=1}^{\infty} \frac{((-1)^n + 3)}{2^{n+1}}; \quad \lim_{n \rightarrow \infty} \frac{1+3}{2^{n+1}} = \frac{2}{2^{n+1}} = \lim_{n \rightarrow \infty} \frac{2 \cdot 2^2}{2^{n+1}} = 8.$$

пр. Даламбера

9/p
2552

$$\sum_{n=1}^{\infty} (\sqrt{n+2} - 2\sqrt{n+1} + \sqrt{n}) = \sqrt{3} - 2\sqrt{2} + \sqrt{1} + \sqrt{4} - 2\sqrt{3} + \sqrt{2} + \sqrt{5} - 2\sqrt{4} + \sqrt{3} + \dots$$

$$\sum_{n=1}^k (\sqrt{n+2} - 2\sqrt{n+1} + \sqrt{n}) = 1 - \sqrt{2} + \sqrt{k+2} - 2\sqrt{k+1} + \sqrt{k}$$

$$\sum_{n=1}^k \sqrt{n+2} - 2 \sum_{n=1}^k \sqrt{n+1} + \sum_{n=1}^k \sqrt{n} = \sqrt{k+2} + \sum_{n=1}^{k-1} \sqrt{n+2} - 2 \sum_{n=1}^{k-1} \sqrt{n+1} + 1 + \sum_{n=1}^{k-1} \sqrt{n+2} =$$

$$-2\sqrt{2} - \sum_{n=1}^{k-1} \sqrt{n+2} + 1 + \sum_{n=1}^{k-1} \sqrt{n+2} \quad \Rightarrow 1 - 2\sqrt{2} + \sqrt{k+2}$$

2553

$\sum_{n=1}^{\infty} \sin(nx)$ 25-6 (невозможно $\sin x \rightarrow 0$ при $n \rightarrow \infty$) "Верно".

2562

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots + \frac{1}{(2n-1)^2} + \dots \quad \sqrt[n]{a_n} = \sqrt[n]{\frac{1}{(2n-1)^2}} \rightarrow 0 = q < 1 \Rightarrow \text{сход.}$$

2567

a) $\sum_{n=1}^{\infty} \min(a_n, b_n)$ может расхожиться; может сходиться $\frac{1}{n^2}$ и $\frac{1}{n^2}$
 б) $\sum_{n=1}^{\infty} \max(a_n, b_n)$; $\max(a_n, b_n) \geq a_n \Rightarrow$ не рас-е. $\frac{1}{n^2}$ и $\frac{1}{n^2}$

2568

$$|a_{n+p} + \dots + a_{n+1}| = (a_{n+p} + \dots + a_{n+1})^2 + \sum_{i,j} a_i a_j \rightarrow 0 \quad \lim_{n \rightarrow \infty} a_n \rightarrow 0$$

Пример $\sum_{k=1}^{\infty} \frac{1}{k^2}$

2575

$$1. \frac{\cos x - \cos 2x}{1} + \frac{\cos 2x - \cos 3x}{2} + \dots + \frac{\cos nx - \cos(n+1)x}{n} + \dots$$

$$= 2 \sin \frac{x}{2} \left(\sin \frac{x}{2} + \sin \frac{3x}{2} + \dots + \sin \frac{(2n+1)x}{2} \right) = 2 \sin \frac{x}{2} \left(\frac{\sin \frac{(2n+1)x}{2}}{\sin \frac{x}{2}} + \dots \right)$$

$$+ \sin \frac{(2n+1)x}{2} \quad \text{25-5.}$$

$$2. \frac{\cos x}{1^2} + \frac{\cos x}{2^2} + \dots + \frac{\cos x}{n^2} + \dots \quad \left| \frac{\cos x^{n+p}}{(n+p)^2} + \dots + \frac{\cos x^{n+1}}{(n+1)^2} \right| \leq$$

$$< \left| \frac{\cos x^{n+1}}{n+1} - \frac{\cos x^{n+1}}{n+1} + \frac{\cos x^{n+2}}{n+1} - \frac{\cos x^{n+2}}{n+2} + \dots + \frac{\cos x^{n+p}}{n+p-1} - \frac{\cos x^{n+p}}{n+p} \right| = 25-9.$$

Σ2574

$$a) 1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \dots$$

$$b) |S_{3np} - S_{3n}| = \frac{1}{3n+1} + \frac{1}{3n+2} - \frac{1}{3n+3} + \dots + \frac{1}{6n-2} + \frac{1}{6n-1} - \frac{1}{6n}$$

$p=3n$:

$$\geq \frac{1}{3n+3} + \frac{1}{3n+2} - \frac{1}{3n+3} + \dots + \frac{1}{6n} + \frac{1}{6n-1} - \frac{1}{6n} = \frac{1}{3} \left(\frac{1}{n+1} + \dots + \frac{1}{2n} \right) > \frac{1}{3} \left(\frac{1}{2n} + \dots + \frac{1}{n} \right)$$

$> \frac{1}{6}$. т.е.

$$d) \frac{1}{\sqrt{1+2}} + \frac{1}{6 \cdot 9} + \dots + \frac{1}{n(n+1)^2}; |S_{3np} - S_n| = \frac{1}{(n+1)\sqrt{n+2}} + \frac{1}{6(n+2)(n+3)} + \dots + \frac{1}{6n(2n+1)} > \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n+1} > \frac{1}{4}$$

Признак Даламбера, Коши,
Раббе, Рунса

~~Σ2575~~ Приз. попул. $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$; Даламбер $\rightarrow$ Раббе

Р. Коши $a_n \geq 0$ $\overline{\lim} \frac{1}{a_n} \leq q < 1$, то сходя.

$$R_n = n \left(1 - \frac{a_n}{a_{n+1}} \right) \rightarrow \dots$$

$\overline{\lim} \frac{1}{a_n} \geq 1$ то расходя
 $\lim \frac{1}{a_n} = q < 1$ - сходя, $q \geq 1$ - расходя

Коши-Мажорант: $a_n, 0 \leq a_{n+1}; f(x)$ - неубыв.

$$\sum \frac{1}{n} = +\infty; \sum_{n=1}^{\infty} \frac{1}{K \ln n}; \lim_{A \rightarrow \infty} \int_{-\frac{1}{2}}^A \frac{dx}{K \ln x} = \lim_{A \rightarrow \infty} \int_{-\frac{1}{2}}^A \frac{d \ln x}{\ln x} = \left| \ln x = t \right| =$$

$$= \lim_{A \rightarrow \infty} \int_{\ln(-1/2)}^{\ln A} \frac{dt}{t} = \lim_{A \rightarrow \infty} (\ln \ln A - \ln \ln(-1/2)) = +\infty$$

$$\sum_{k=1}^{\infty} \frac{1}{k^2} \int_1^{\infty} \frac{dx}{x^2} \quad \alpha > 1 \text{ сходя.}$$

$0 < \alpha \leq 1$ расходя

Σ2581

$$\sum_{n=1}^{+\infty} \frac{2^n n!}{n^n}; \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{2^{n+1} (n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n n!} = \lim_{n \rightarrow \infty} \frac{2(n+1) \cdot n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} 2 \left(\frac{n}{n+1} \right)^n = \frac{2}{e} < 1$$

2581 $\sum_{n=1}^{\infty} \frac{3^n \cdot n!}{n^n}$ $\lim_{n \rightarrow \infty} \frac{3^{n+1} (n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{3^n \cdot n!} = \lim_{n \rightarrow \infty} 3 \left(\frac{n}{n+1}\right)^n = 3 \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1}\right)^n = \frac{3}{e} > 1$
 $\rightarrow$ $\sum$ $\rightarrow$ ∞ .

2582 $\sum_{n=1}^{\infty} \frac{(n!)^2}{2^{n^2}}$ $\lim_{n \rightarrow \infty} \frac{((n+1)!)^2}{2^{(n+1)^2}} \cdot \frac{2^{n^2}}{(n!)^2} = \lim_{n \rightarrow \infty} \frac{(n+1)^2}{2^{2n+1}} = 0$.

2586 $\sum_{n=1}^{\infty} \frac{n^2}{(2 + \frac{1}{n})^n}$ $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \frac{n^{\frac{2}{n}}}{2 + \frac{1}{n}} = \frac{n^{\frac{2}{n}}}{2 + \frac{1}{n}} \xrightarrow{n \rightarrow \infty} \frac{1}{2} < 1 \rightarrow$ $\sum$ $\rightarrow$ ∞ .

2588 $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ $\lim_{n \rightarrow \infty} \frac{1}{n \ln n} < 1$; $\sqrt[n]{\frac{1}{n \ln n}} = \frac{1}{\sqrt[n]{n \ln n}} < 1$; $\frac{1}{\sqrt[n]{n \ln n}} > \frac{1}{n \ln n} \rightarrow 1$
 $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ - $\rightarrow$ ∞ .

2596 $\sum_{n=1}^{\infty} \frac{n \cos \frac{\pi n}{3}}{2^n}$ $\lim_{n \rightarrow \infty} \frac{\sqrt[n]{n} \cdot \sqrt[n]{\cos \frac{\pi n}{3}}}{2} < \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1$ $\rightarrow$ $\sum$ $\rightarrow$ ∞ .

2597 $\sum_{n=1}^{\infty} \frac{n^3 (\sqrt{2} + (-1)^n)^n}{3^n}$ $\lim_{n \rightarrow \infty} \frac{n^3 (\sqrt{2} + (-1)^n)^n}{3^n} < \lim_{n \rightarrow \infty} \frac{n^3 (1 + \sqrt{2})^n}{3^n}$
 $\lim_{n \rightarrow \infty} \frac{n^3 (1 + \sqrt{2})^n}{3^n} = \frac{1 + \sqrt{2}}{3} < 1 \rightarrow$ $\sum$ $\rightarrow$ ∞ .

2613 $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ $f(x) = \frac{1}{x \ln x}$ $\int_2^{+\infty} \frac{1}{x \ln x} dx = \int_2^{+\infty} \frac{1 \ln x}{\ln x} = \left| \ln x = t \right| = \left| \frac{x}{2} \right| = \frac{x}{2} \Big|_2^{+\infty} = \frac{+\infty}{2} - \frac{2}{2} = +\infty$
 $\rightarrow$ $\sum$ $\rightarrow$ ∞ .

2618 $\sum_{n=1}^{\infty} \left[\frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{2 \cdot 4 \cdot 6 \cdots (2n+2)} \right]^p$ $\lim_{n \rightarrow \infty} \left[\frac{1 \cdot 3 \cdots (2n+3)}{2 \cdot 4 \cdots (2n+4)} \cdot \frac{2 \cdot 4 \cdots (2n+2)}{1 \cdot 3 \cdots (2n+1)} \right]^p = \left(\frac{2n+3}{2n+4} \right)^p$
 $\rightarrow \lim_{n \rightarrow \infty} \left(1 - \frac{1}{2n+4} \right)^p = \lim_{n \rightarrow \infty} \left(1 - \frac{p}{2n+4} + o\left(\frac{p}{2n+4}\right) \right) = 1$
 $\rightarrow$ $\sum$ $\rightarrow$ ∞ .

$$\sum \frac{1}{n^a} \quad a) 1 - \text{cresc.} \quad \frac{a_{n+1}}{a_n} = \frac{1}{(1+\frac{1}{n})^a} = (1+\frac{1}{n})^{-a};$$

$$\frac{p_{n+1}}{p_n} = 1 - \frac{a}{n} + o(\frac{1}{n}) ; \quad \sum p_n, p_n = \frac{1}{n^a} ; \quad \text{No yes-no } a > 1$$

$$a > 1 ; \quad \frac{p_{n+1}}{p_n} = 1 - \frac{a}{n} + o(\frac{1}{n}) ; \quad \frac{p_{n+1}}{p_n} - \frac{p_{n+1}}{p_{n+1}} = \frac{1-n}{n} + o(\frac{1}{n})$$

1/26/21
6/29/21

$$\frac{p_{n+1}}{p_n} > \frac{p_{n+1}}{p_{n+1}}$$

2/25/21

$$\frac{1000}{1} + \frac{1000 \cdot 1000}{1 \cdot 3} + \frac{1000 \cdot 1000 \cdot 1000}{1 \cdot 3 \cdot 5} + \dots$$

$$\frac{a_{n+1}}{a_n} = \frac{1000 + (n-1)}{2n-1} = \frac{1999 + n}{2n-1} \rightarrow \frac{1}{2} < 1$$

2/25/21(2)

$$\sum_{n=1}^{\infty} \frac{n^5}{2+3^n}$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^5}{2^{n+1}+3^{n+1}} \cdot \frac{(2^n+3^n)}{(n)^5} = \frac{(1+\frac{1}{n})^5 (2+\frac{3}{2})^n}{2(\frac{2}{3})^n + 1} \rightarrow 1$$

$$\lim_{n \rightarrow \infty} \frac{n^5}{2+3^n} < \lim_{n \rightarrow \infty} \frac{n^5}{2^{n+1}} = \frac{1}{2} \Rightarrow \text{cresc.}$$

2/25/21(1)

2/25/21

$$\frac{a}{b} + \frac{a(a+b)}{b(b+d)} + \frac{a(a+b)(a+2b)}{b(b+d)(b+2d)} + \dots, \quad a > 0, b > 0, d > 0.$$

$$\frac{a_{n+1}}{a_n} = \frac{b + (n-1)d}{a + (n-1)d} ; \quad \lim_{n \rightarrow \infty} n \left(1 - \frac{a_{n+1}}{a_n} \right) = \lim_{n \rightarrow \infty} n \left(\frac{a + (n-1)d - b - (n-1)d}{a + (n-1)d} \right) =$$

$$= \lim_{n \rightarrow \infty} \frac{a-b}{a + (n-1)d} = \frac{a-b}{d} > 1 \Rightarrow \text{cresc.}$$

$$< 1 \Rightarrow \text{pauc.}$$

2/25/21

$$\sum_{n=1}^{\infty} \frac{n! e^n}{n^{n+p}} ; \quad \frac{a_{n+1}}{a_n} = \frac{(n+1)e}{(n+1)^{n+p+1}} \cdot \frac{n^{n+p}}{1} = e \left(\frac{n}{n+1} \right)^{n+p} = e \left(1 - \frac{1}{n+1} \right)^{n+p} \rightarrow \left(\frac{1}{e} \right)$$

$$\lim_{n \rightarrow \infty} n \left(1 - e \left(1 - \frac{1}{n+1} \right)^{n+p} \right) ; \quad \frac{1}{e \left(1 - \frac{1}{n+1} \right)^{n+p}} = \frac{1}{e} \left(1 - \frac{1}{n+1} \right)^{-n-p} =$$

$$= \frac{1}{e} \left(1 + \frac{n+p}{n+1} + \frac{(-n-p)(-n-p-1)}{2} \frac{1}{(n+1)^2} + o\left(\frac{1}{n^2}\right) \right) = \frac{1}{e} \left(1 + 1 + \frac{p-1}{n+1} + \dots \right)$$

$$\lim_{n \rightarrow \infty} n \left(\frac{a_n}{a_{n+1}} - 1 \right) = \lim_{n \rightarrow \infty} \frac{\frac{1}{e} \left(\frac{n+1}{n} \right)^{n+p} - 1}{\frac{1}{n}} = \lim_{x \rightarrow 0} \frac{\frac{1}{e} (1+x)^{\frac{1}{x}+p} - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{1}{e} e^{(1+p)/e} - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{e} e^{1 + (p - \frac{1}{2})x + o(x)} - 1}{x} = p - \frac{1}{2} \quad p - \frac{1}{2} > 1 \quad p > \frac{3}{2} \text{ conv.}$$

2610

$$a) \sum_{n=1}^{\infty} \frac{\ln 2 \cdot \ln 3 \cdot \dots \cdot \ln(n+1)}{\ln(2+p) \cdot \ln(3+p) \cdot \dots \cdot \ln(n+p)} \quad (p > 0)$$

$$\frac{a_{n+1}}{a_n} = \frac{\ln(n+1)}{\ln(n+p)} = \ln(n+1) - \ln(n+p) < \ln(p) \quad \log_{n+p}(n+1) < 0$$

~~$$\int_1^{\infty} \frac{\ln 2 \cdot \ln 3 \cdot \dots \cdot \ln(x+1)}{\ln(2+p) \cdot \ln(3+p) \cdot \dots \cdot \ln(x+p)} dx$$~~

$$\int_1^{\infty} \frac{\ln(x+1)}{\ln(x+p)} dx \rightarrow \int_1^{\infty} \ln \log \frac{x+1}{x+p} dx$$

2620

$$i) \sum_{n=1}^{\infty} \frac{v(n)}{n^2}, \quad v(n) = \lceil \log_{10} n \rceil; \quad \int_2^{\infty} \frac{\lceil \log_{10} x \rceil}{x^2} dx < \int_2^{\infty} \frac{\log_{10} x + 1}{x^2} dx = \int_2^{\infty} \frac{\log_{10} x}{x^2} dx + \int_2^{\infty} \frac{1}{x^2} dx$$

$$F(\log_{10} x) := \frac{1}{3} \frac{\log_{10} x}{x^3}; \quad \Rightarrow \text{conv.};$$

$$\frac{a_{n+1}}{a_n} = \frac{\log_{10} n+1}{(n+1)^2} \cdot \frac{n^2}{\log_{10} n} = \frac{\log_{10}(n+1)}{\log_{10} n \cdot (1 + \frac{1}{n})^2} \rightarrow 1$$

$$\lim_{n \rightarrow \infty} n \left(1 - \frac{\log_{10}(n+1)}{\log_{10} n \cdot (1 + \frac{1}{n})^2} \right) = \lim_{n \rightarrow \infty} \frac{\log_{10} n}{\log_{10} n \cdot (1 + \frac{1}{n})^2} = 1$$

$$\text{conv.} < \int_2^{\infty} \frac{\log_{10} x}{x^2} dx = \frac{1}{\ln 10} \frac{\log_{10}^2 x}{2} \Big|_2^{\infty} = \frac{1}{2 \ln 10}$$

2620

$$i) \sum_{n=2}^{\infty} \frac{1}{n \ln(n)}; \quad \int_2^{\infty} \frac{1}{x \ln x} dx \quad \ln(n!) < n \ln n \quad \frac{1}{\ln(n!)} > \frac{1}{n \ln n} > 0$$

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n} - pae - u \Rightarrow pae - u.$$

2614.1



$$a_n > 0; \quad \sum_{n=1}^{\infty} a_n \text{ с.с. } \exists (1-\sqrt[n]{a_n})^n \geq p > 1 \text{ при } n > n_0$$

$$\sum_{n=1}^{\infty} a_n \text{ расхожд. } (1-\sqrt[n]{a_n})^n \leq 1 \text{ при } n > n_0.$$

$$0 \leq a_n \leq (1 - \frac{p \ln n}{n})^n, n > n_0 \quad 1 - \frac{p \ln n}{n} > 0 \Rightarrow 0 \leq a_n \leq \exp \{ n \ln (1 - \frac{p \ln n}{n}) \}$$

$$\text{т.к. } a_n \leq \frac{1}{n^p} \exp \{ -p \frac{\ln^2 n}{2n} + o(\frac{\ln^2 n}{n}) \} = \frac{1}{n^p} - p \frac{\ln^2 n}{2n^{p+1}} + o(\frac{\ln^2 n}{n^{p+1}}), n \rightarrow \infty$$

e) $p > 1$ с.с.; д.с.

Аналогично, $a_n \geq \frac{1}{n} - \frac{\ln^2 n}{2n^2} + o(\frac{\ln^2 n}{n^2}) = O^*(\frac{1}{n}), n \rightarrow \infty$ т.е. расх.

2615

$$a_n > 0; \quad \sum_{n=1}^{\infty} a_n; \quad \exists \alpha > 0: \frac{\ln \frac{1}{a_n}}{\ln n} \geq 1 + \alpha \text{ при } n \geq n_0 \Rightarrow \text{расхожд.}$$

$$\frac{\ln \frac{1}{a_n}}{\ln n} \leq 1 \text{ при } n \geq n_0 \Rightarrow \text{с.с.}$$

Из условия получаем: $a_n \leq \frac{1}{n^{1+\alpha}}$ и $a_n \geq \frac{1}{n}$ при $n \geq n_0$ д.с.

Числ. ряд

$$\sum_{n=1}^{\infty} \left(\frac{(2n-1)!}{(2n)!} \right)^p; \quad p=2 \quad \frac{a_n}{a_{n+1}} = \left(1 + \frac{1}{2n+1} \right)^2 = 1 + \frac{2}{2n+1} + o(\frac{1}{n})$$

$$\frac{1}{2n+1} = \frac{1}{2n(1+\frac{1}{2n})} = \frac{1}{2n} \cdot \frac{1}{1+\frac{1}{2n}} \geq \frac{1}{2n} \left(1 - \frac{1}{2n} + o(\frac{1}{n}) \right) = \frac{1}{2n} - \frac{1}{4n^2} + o(\frac{1}{n^2})$$

$$\frac{1}{1+x} = 1 - x + o(x^2); \quad \ominus \quad 1 + \frac{1}{n} - \frac{1}{2n^2} + o(\frac{1}{n^2}) = 1 + \frac{1}{n} + o(\frac{1}{n}), \quad \lambda=1, \quad \theta=1$$

$$a_n = \left(\frac{1 \cdot 3 \cdot \dots \cdot (2n-1)}{2 \cdot 4 \cdot \dots \cdot 2n} \right)^2 = \frac{n!}{(2n)!} = \frac{2^n n!}{(2n)!}$$

$$\approx \left(\frac{1}{2^2} \right) \left(\frac{1}{2^2} \right) \dots \left(\frac{1+\frac{1}{n}}{2^2} \right)^2; \quad n! \sim \left(\frac{n}{e} \right)^n \sqrt{2\pi n}$$

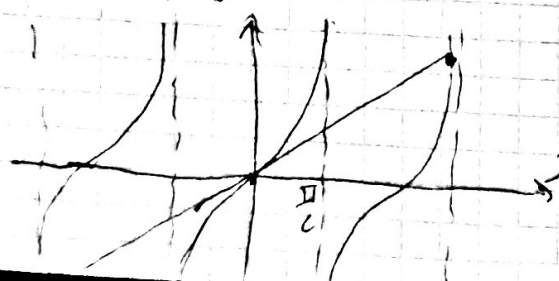
Тогда $a_n \sim \left(\frac{(n/e)^n \sqrt{4\pi n}}{2^{2n} (n/e)^n} \right)^2 = \frac{4\pi n}{4\pi n^2} = \frac{1}{n} = O^*(\frac{1}{n}) \Rightarrow \text{расх.-с.с.}$

1) $\lambda > 1$ с.с.
 2) $\lambda < 1 \Rightarrow$ расх.
 3) $\lambda = 1$
 а) $\theta > 1$ с.с.
 б) $\theta \leq 1$ расх.

2621

числ. ряд
 λ_n — корни уравнения

$\lg x = x$ (ср-ге и п-ге) тогда $\sum_{n=1}^{\infty} \frac{1}{\lambda_n}$



$$\lambda_n \approx \frac{n}{2} + \sqrt{n}, \quad \frac{n}{2} + (n-2)\sqrt{n} < \lambda_n < \frac{n}{2} + (n-1)\sqrt{n}$$

$$\frac{n}{2} + \sqrt{n} < \lambda_n < \frac{n}{2} + \sqrt{n}$$

$$\frac{1}{x^2} = O\left(\frac{1}{n^2}\right); \quad f(x) = O^*(g(x)) \Leftrightarrow f(x) = O(g(x)) \\ (g(x) = O(f(x))) \\ \Rightarrow \text{сходится.}$$

Р. 30.
 $|f(x)| < c|g(x)|$

а) $f(x) \rightarrow 0$ при $x \rightarrow a$
 $f(x) = x(x+x^2)$

Пр. Ред. $\sum_{n=1}^{\infty} a_n$ не-абс. абс. x , или
 эк. $\sum |a_n|$. Если сход., то абс. $\Rightarrow$ ун. сход.

Ред. (признак Лейбница)

Пусть $\{a_n\}$ монотон. стрем. к 0. Тогда ред. $\sum_{n=1}^{\infty} (-1)^n a_n$ эк.

$$\sum_{n=1}^{\infty} u_n v_n;$$

	ун. на u_n	ун. на v_n
признак Дирихле	ЖС: $\sum_{k=1}^n u_k \leq c$	v_n монотонно $\rightarrow 0 \Rightarrow$ эк.
признак Абеля	$\sum_{n=1}^{\infty} u_n$ эк.	v_n монотонно и стр. $\Rightarrow$ эк.

$$S_n = \sum_{k=1}^n \sin kx, \quad T_n = \sum_{k=1}^n \cos kx$$

$$|S_n, T_n| \leq \frac{1}{|\sin \frac{x}{2}|}$$

22701

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \text{ эк. по пр. Дирихле}$$

$$S_n = \sum_{k=1}^n \frac{(-1)^k}{\sqrt{k}} + \sum_{k=1}^n \frac{1}{k} \left(\sum_{k=1}^n \frac{1}{k} \right) \quad \forall A > 0.$$

2666

$$\sum_{n=1}^{\infty} (-1)^n b_n, \quad b_n > 0, b_n \rightarrow 0 (n \rightarrow \infty); \text{ Ред. эк. 1; } n$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{2+(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{2(-1)^n}{n} + \frac{1}{n}; \quad S_N = \sum_{n=1}^N \frac{2(-1)^n + 1}{n} = \sum_{n=1}^N \frac{2(-1)^n}{n} + \sum_{n=1}^N \frac{1}{n}$$

эк. $c + \ln n + o(1)$
 $\Downarrow$
 расхож.

2665(a, b)

а) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2n+100}{3n+1} \right)^n$ эк., избегать

б) $1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{5} - \frac{1}{6} + \frac{1}{7} + \frac{1}{8} - \dots$

$$S_n = \sum_{k=1}^n \frac{x_k}{k}, \quad x_k = \begin{cases} 1, & k \bmod 6 = 1, 2, 5 \\ -1, & k \bmod 6 = 3, 4, 6 \end{cases}, \quad S_n = S_{6m} + \dots$$

$$S_{6n} = \left(1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{5} - \frac{1}{6} \right)_n$$

$$S_{6n+1} = S_{6n} + \frac{1}{6n+1} \rightarrow 0 \quad S_{6n+5} = S_{6n} + \frac{1}{6n+1} + \frac{1}{6n+2} + \frac{1}{6n+3} - \frac{1}{6n+4} -$$

$$S_{6n+6} < S_{6n} + \frac{1}{4n^2}$$

$$\frac{1}{6n-5} + \frac{1}{6n-5} + \frac{1}{6n-3} - \frac{1}{6n-2} - \frac{1}{6n-1} - \frac{1}{6n} = \frac{3}{(6n-5)(6n-4)} + \frac{3}{(6n-4)(6n-1)} + \frac{3}{(6n-3)(6n)}$$

$O'(100)$

2659

$$1 - \frac{3}{2} + \frac{5}{4} - \frac{7}{8} + \dots + \frac{2n+1}{2n} - \frac{2n+3}{2n+2} + \dots$$

$$\frac{4n+2-2n-3}{4n} = \frac{2n-1}{4n}$$

$$b_n = \frac{2n+1}{2n} \rightarrow 0$$

$$S_{2n} = \sum_{k=1}^n \frac{2k-1}{4k}, \quad S_{2n+1} = S_{2n} + \frac{2(n+1)}{2n+2}$$

$$\rightarrow \frac{1}{2} \quad \rightarrow \frac{1}{2} + 1 = \frac{3}{2}$$

$$S_n = \sum_{k=1}^n (-1)^{k-1} \frac{2k-1}{2^{k-1}}$$

$$\frac{1}{2} S_n = \frac{1}{2} - \frac{3}{2^2} + \frac{5}{2^3} - \dots + (-1)^{n-1} \frac{2n-3}{2^{n-1}} + (-1)^{n-1} \frac{2n-1}{2^n}$$

$$\frac{3}{2} S_n = 1 - \frac{2}{2} + \frac{2}{2^2} - \frac{2}{2^3} + \dots + (-1)^{n-1} \frac{2}{2^{n-1}} + (-1)^{n-1} \frac{2n-1}{2^n}$$

$$3S_n = 2 - 2 + \frac{2}{2} - \frac{2}{2^2} + \dots + (-1)^{n-1} \frac{2}{2^{n-2}} + (-1)^{n-1} \frac{2n-1}{2^{n-1}} = \frac{1 - (-\frac{1}{2})^{n-2}}{1 - (-\frac{1}{2})} + (-1)^{n-1} \frac{2n-1}{2^{n-1}} \rightarrow \frac{2}{3}$$

$$\Rightarrow \lim_{n \rightarrow \infty} S_n = \frac{2}{3}$$

2661

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \dots$ по пр. действующая серия

$$S_{2n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2n-1} - \frac{1}{2n} = (1 + \frac{1}{2} + \dots + \frac{1}{2n}) - 2(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2n}) =$$

$$= C + \ln 2n + \varepsilon_n - 2 \cdot \frac{1}{2} (C + \ln n + \varepsilon_n) = \ln 2 + \varepsilon_n - \varepsilon_n = \ln 2 + O(\frac{1}{n})$$

$$\lim_{n \rightarrow \infty} S_{2n} = \ln 2, \quad \lim_{n \rightarrow \infty} S_{2n+1} = \lim_{n \rightarrow \infty} (S_{2n} + \frac{1}{2n+1}) = \ln 2 \Rightarrow \lim_{n \rightarrow \infty} S_n = \ln 2$$

2702(05)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p} \quad p > 0, \quad S \in [\frac{1}{2}, 1] \quad S_n = 1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots + \frac{(-1)^{n+1}}{n^p}$$

$$\sum_{n=1}^{\infty} (\frac{1}{n^p} - \frac{1}{(n+1)^p}) ; \quad \frac{1}{n^p} - \frac{1}{(n+1)^p} = \frac{1}{n^p} (1 - \frac{1}{(1+\frac{1}{n})^p}) = \frac{1}{n^p} (1 - (1 - \frac{p}{n} + O(\frac{1}{n^2}))) = \frac{p}{n^{p+1}} + O(\frac{1}{n^{p+2}})$$

т.к. $a_n = \frac{1}{n^p} - \frac{1}{(n+1)^p}$ - мон. число и $a_n \rightarrow 0$ при $n \rightarrow \infty$, то $\sum_{n=1}^{\infty} a_n$ сход. по пр. действующая серия.

$\Rightarrow$ мон. числ. и $a_n \rightarrow 0$ не нужны.

2678

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n \sin^n x}{n} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(1 - \cos 2x)^n}{n}$$

по пр. действующая серия; по пр. Дале. $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = 0$

$$S_{2n} = (1 - \frac{1}{2^p}) + (\frac{1}{3^p} - \frac{1}{4^p}) + \dots + (\frac{1}{(2n-1)^p} - \frac{1}{(2n)^p})$$

$$S_{2n+1} = 1 - (\frac{1}{2^p} - \frac{1}{3^p}) - \dots - (\frac{1}{(2n-1)^p} - \frac{1}{(2n)^p}) \Rightarrow S_{2n} < S < S_{2n+1}$$

$$\Rightarrow S < S_1 < 1$$

Рассмотрим S_{n-1} , $\forall p > 0, x > 0$

$\lim_{n \rightarrow \infty} \left(\frac{1}{p} + \frac{1}{p} > \frac{2}{p}, \frac{1}{p} + \frac{1}{p} > \frac{2}{p}, \dots, \frac{1}{(n-1)^p} + \frac{1}{(n+1)^p} > \frac{2}{(n)^p} \right) \Rightarrow$
 $\Rightarrow \sum_{k=1}^n \frac{1}{k^p} = 1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots + \frac{1}{(n-1)^p} - \frac{1}{(n)^p} + \frac{1}{(n+1)^p} > 1 - \frac{1}{2^p} + \frac{1}{4^p} - \dots - \frac{1}{(n-2)^p} + \frac{1}{(n)^p} =$
 $= 1 - \frac{1}{2^p} \sum_{k=1}^n \frac{1}{k^p} \Rightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k^p} = S \geq 1 - \frac{1}{2^p} \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k^p} = 1 - \frac{S}{2^p}, S \geq \frac{2^p}{2^p+1} > \frac{1}{2} \quad \forall p > 0.$

2671

$\sum_{n=1}^{\infty} \sin(\pi \sqrt{n^2 + k^2}), \quad \sin(\pi \sqrt{n^2 + k^2}) = \sin(\pi k \sqrt{1 + (\frac{k}{n})^2}) = \sin(\pi k (1 + \frac{k^2}{2n^2} + O(\frac{1}{n^4}))) =$
 $= \sin(\pi k + \frac{k^3 \pi}{2n^2} + O(\frac{1}{n^3})) = (-1)^n \sin(\frac{k^3 \pi}{2n^2} + O(\frac{1}{n^3})) = (-1)^n \frac{k^3 \pi}{2n^2} + O(\frac{1}{n^3}).$
 $\sum_{k=1}^{\infty} (-1)^n \frac{k^3 \pi}{2n^2}$ сходится по пр. Лебнесса, $\sum_{n=1}^{\infty} \frac{1}{n^3}$ сходится по кресту. пр. Коши.
 Остаток $o(n^{-2})$.

2672

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}, \quad \text{Рассмотрим ряд } \sum_{n=1}^{\infty} (-1)^n A_n, \quad A_n = \frac{1}{(n^2)^p} + \frac{1}{(n^2+1)^p} + \dots + \frac{1}{(n^2+2n)^p}.$
 $0 < A_n < \frac{2n+1}{n^2} \rightarrow 0, n \rightarrow \infty \text{ и } A_n - A_{n+1} = \sum_{i=0}^{2n} \frac{(i+1)^2 + 1 - (n^2+1)}{(i^2+1)(n^2+2n+i+1)} = \frac{(n^2+4n+2) - (n^2+1)}{(i^2+1)(n^2+2n+i+1)} =$
 $= \frac{(2n+1)(n^2+2n+1) - (n^2+1)}{(i^2+1)(n^2+2n+i+1)} = \frac{1}{(n^2+2n+1)} - \frac{1}{(n^2+2n+i+1)} > 0$ при пр. Далам. и
 $\Rightarrow$ по пр. Лебнесса $\sum_{n=1}^{\infty} (-1)^n A_n$ сх. $\Rightarrow$ исходный ряд сх.

2673(a, b)

a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}, \quad \left(\frac{1}{(n+1)^p} \right)^{-1} \cdot \frac{1}{n^p} = \frac{n^p}{(n+1)^p} \rightarrow 1 \Rightarrow$ критерий ун.
 б) $\sum_{n=2}^{\infty} \frac{1}{n^p} \cos \frac{\pi n^2}{n+1}, \quad \cos \frac{\pi n^2}{n+1} = (-1)^n \cos \left(\pi \frac{n^2}{n+1} - \pi n \right) = (-1)^n \cos \frac{\pi}{n+1}.$
 По пр. Лебнесса сх., а по пр. Далам. и пр. Коши.

2674

$b_1 - b_2 + b_3 - b_4 + \dots + (-1)^{n-1} b_n + \dots \quad (b_n > 0) \text{ сх. , если } \frac{b_n}{b_{n+1}} = 1 + \frac{p}{n} + O(\frac{1}{n^2}), p > 0.$
 Рассм. абсол. сх.: $\varepsilon > 0$ произ-вольно, $b_n > 0 \Rightarrow n \rightarrow \infty \quad b_n = O(\frac{1}{n^{p+\varepsilon}})$
 По пр. Лебнесса сх.

2676

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{p+1}}, \quad p=0 \Rightarrow$ ряд сх.; $p > 0$ пр. Абсол. сх. $\Rightarrow \frac{1}{n^{p+1}}$
 А при $p > 1$: $0 < p \leq 1$: $\frac{(-1)^{n-1}}{n^p} \cdot \frac{1}{n}$ пр. Абсол. сх. $\Rightarrow$ пр. Коши и пр. Далам.
 $\Rightarrow p > 1$ сх., т.к. $a_n \sim O(\frac{1}{n^p})$; $p < 0$ ряд сх.

Σ 203a

$$a) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$$

Σ 2704

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots (-1)^{n-1} \frac{1}{n} ; S_{p+q} = 1 + \frac{1}{2p-1} + \frac{1}{2} \left(1 + \frac{1}{2} + \dots + \frac{1}{2q} \right) =$$

$$= \frac{1}{2} \left(2 + \frac{1}{p} - \ln q + \varepsilon \right) ; S_{n(p+q)} = \ln(2p-1) - \ln nq + \varepsilon = \ln \left(\frac{2p}{q} - \frac{1}{nq} \right) + \varepsilon$$

$$\Rightarrow \sum_{n,p} = \ln 2 + \ln \frac{p}{q}$$

Σ 2608

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \cdot \sin \frac{\pi}{n} \quad \sin x < x, x \in (0, \frac{\pi}{2})$$

$$p > 0. \quad \frac{\pi}{n} > \sin \frac{\pi}{n} > 0 \quad \Leftrightarrow \pi \sum_{n=1}^{\infty} \frac{1}{n^{p+1}} < \infty, \text{ wenn } p+1 > 1, p > 0$$

Σ 2609

$$\sum_{n=2}^{\infty} (\sqrt{n+1} - \sqrt{n})^p \ln \frac{n+1}{n+1} = \sum_{n=2}^{\infty} \frac{1}{(\sqrt{n+1} + \sqrt{n})^p} \ln \left(1 - \frac{2}{n+1} \right) = \sum_{n=2}^{\infty} \frac{1}{(\sqrt{n+1} + \sqrt{n})^p} \left(-\frac{2}{n+1} + O\left(\frac{1}{n^2}\right) \right)$$

$$u_n \sim \frac{1}{n^{p/2+1}}, p/2+1 > 1 \Rightarrow \text{div.}$$

Σ 2612

$$\sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} \left(e - \left(1 + \frac{1}{n} \right)^n \right)^p = \sum_{n=1}^{\infty} \left(e - e^{n \ln(1 + \frac{1}{n})} \right)^p = \sum_{n=1}^{\infty} \left(e - e^{n \left(\frac{1}{n} - \frac{1}{2n^2} + O\left(\frac{1}{n^3}\right) \right)} \right)^p =$$

$$= \sum_{n=1}^{\infty} \left(e - e^{1 - \frac{1}{2n} + O\left(\frac{1}{n^2}\right)} \right)^p = \sum_{n=1}^{\infty} \left(e - e^{1 - \frac{1}{2n} + \frac{1}{8n^2} + O\left(\frac{1}{n^3}\right)} \right)^p = \sum_{n=1}^{\infty} \left(\frac{e}{e^{1 - \frac{1}{2n} + \frac{1}{8n^2} + O\left(\frac{1}{n^3}\right)}} \right)^p = \sum_{n=1}^{\infty} \frac{e^p}{(2n)^p} \cdot \frac{1}{n^p}$$

$$e^x = 1 + x + \frac{x^2}{2!} + O(x^3)$$

Σ 2626

$$\sum_{n=3}^{\infty} \frac{\sqrt{n+2} - \sqrt{n-2}}{n^x} = \sum_{n=3}^{\infty} \frac{4}{(\sqrt{n+2} + \sqrt{n-2}) n^x}, u_n \sim \frac{1}{n^{x+\frac{1}{2}}} \quad \text{cx, wenn } x > \frac{1}{2}$$

$$\alpha \in \frac{1}{2} \text{ p. ex.}$$

Σ 26

$$\sum_{n=1}^{\infty} \sqrt{n+a} - \sqrt{n+b} = \sum_{n=1}^{\infty} \frac{(n+a)^2 - (n+b)^2}{(\sqrt{n+a} + \sqrt{n+b})(n+a + \sqrt{n^2+nb})}$$

$$\text{Term } n: a-b = \frac{a^4 - b^4}{(a+b)(a^2+b^2)}$$

$$a \neq \frac{1}{2} \text{ пар. } u_n \sim O^*\left(\frac{1}{n^{\frac{1}{2}}}\right); \quad a = \frac{1}{2} \Rightarrow u_n \sim O^*\left(\frac{1}{n^{\frac{1}{2}}}\right) \text{ нек.}$$

$$\sqrt{2629} \quad \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \sqrt{\ln \frac{n+1}{n}} \right)$$

$$\frac{1}{n+1} < \ln \left(1 + \frac{1}{n} \right) < \frac{1}{n};$$

$$\ln \left(\frac{n+1}{n} \right) = -\ln \left(\frac{n}{n+1} \right) = -\ln \left(1 - \frac{1}{n+1} \right) > \frac{1}{n+1} \text{ по разл.}$$

$$\left(\frac{1}{\sqrt{n}} - \sqrt{\ln \frac{n+1}{n}} \right) < \frac{1}{\sqrt{n}} - \frac{1}{n+1} = \frac{\sqrt{n} - \sqrt{n+1}}{\sqrt{n} \sqrt{n+1}} = \frac{1}{\sqrt{n} \sqrt{n+1} (\sqrt{n+1} + \sqrt{n})} \Rightarrow \text{ск. по}$$

критер. по Мажор.
Корн.

$\sqrt{2636}$

$$\sum_{n=1}^{\infty} \left(\cos \frac{a}{n} \right)^{n^2}; \quad \sqrt[n]{a_n} = \left(\cos \frac{a}{n} \right)^n; \quad \cos \frac{a}{n} = 1 - \frac{a^2}{2n^2} + O\left(\frac{1}{n^4}\right)$$

$$\sqrt[n]{a_n} = \left(1 - \frac{a^2}{2n^2} + O\left(\frac{1}{n^4}\right) \right)^n; \quad \left(\cos \frac{a}{n} \right)^{n^2} = e^{n^2 \ln \cos \frac{a}{n}} = e^{n^2 \ln \left(1 - \frac{a^2}{2n^2} + O\left(\frac{1}{n^4}\right) \right)} =$$

$$= e^{n^2 \left(-\frac{a^2}{2n^2} + \frac{a^4}{24n^4} + O\left(\frac{1}{n^6}\right) \right)} = e^{-\frac{a^2}{2} + O\left(\frac{1}{n^2}\right)} \Rightarrow \text{прогр.}$$

$$\ln(1+x) = x - \frac{x^2}{2} + O(x^3) \quad \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = e^{-\frac{a^2}{2}} < 1$$

$\sqrt{2639}$

$$\sum_{n=2}^{\infty} \frac{n \ln n}{(\ln n)^n} \quad a_n = \frac{n \ln n}{(\ln n)^n} = \frac{e^{\ln n}}{e^{n \ln \ln n}} = e^{\ln n - n \ln \ln n}$$

$$\lim_{n \rightarrow \infty} \frac{\ln^2 n - n \ln \ln n}{n^a} = \lim_{n \rightarrow \infty} \frac{2 \ln n \cdot \frac{1}{n} - (\ln \ln n + n \cdot \frac{1}{n} \cdot \frac{1}{n})}{a n^{a-1}} = 0 \Rightarrow O^*\left(\frac{1}{n}\right) - \text{пар. по}$$

$$R_n = n \left(\frac{u_n}{u_{n+1}} - 1 \right) \rightarrow q, \quad q > 1 - \text{ск.}$$

$$R_n = n \left(1 - \frac{u_{n+1}}{u_n} \right) \rightarrow q, \quad q > 1 - \text{ск.}$$

$$\sum_{n=1}^{\infty} p'_n, \quad p \frac{u_{n+1}}{u_n} = 1 - \frac{b}{n} + O\left(\frac{1}{n^2}\right), \quad b > 1 \Rightarrow \text{ск.} \quad \frac{u_{n+1}}{u_n} = f\left(\frac{1}{n}\right); \quad \text{прогр.}$$

$$\sum_{n=0}^{\infty} (c_n - (-1)^n); \quad C_{2m} = \underbrace{(c_1 - c_2)}_{\geq 0} + \underbrace{(c_2 - c_3)}_{\geq 0} + \dots + \underbrace{(c_{2m-1} - c_{2m})}_{\geq 0}$$

$$C_{2m} = c_1 - (c_2 - c_3) - \dots - (c_{2m-2} - c_{2m-1}) - c_{2m} < c_1$$

$$\Rightarrow \lim C_{2m} = a; \quad C_{2m+1} = C_{2m} + c_{2m+1} \rightarrow a \quad \text{прогр.}$$

$$f''(0) + \frac{1}{n} \left(\frac{f''(0)}{n} \right) + O\left(\frac{1}{n^2}\right), \quad f(0) = 1, \quad f'(0) < -1$$

$$2 \sin 2 \sin \theta = \cos(2-\theta) - \cos \theta$$

$\sqrt{2667}$

$$\sum_{n=1}^{\infty} \frac{\ln n}{n} \cdot \sin \frac{\pi n}{4} \quad \lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0 \quad \frac{1}{n} \sum_{k=1}^n 2 \sin \frac{\pi k}{4} \sin \frac{\pi}{8} = \frac{1}{2 \sin \frac{\pi}{8}} \left(\sum_{k=1}^n \cos \left(\frac{\pi k}{4} - \frac{\pi}{8} \right) + \cos \left(\frac{\pi}{4} \right) \right)$$

$$= \frac{1}{2 \sin \frac{\pi}{8}} \left(\cos \left(\frac{\pi}{4} - \frac{\pi}{8} \right) - \cos \left(\frac{\pi}{4} + \frac{\pi}{8} \right) + \cos \left(\frac{3\pi}{4} - \frac{\pi}{8} \right) - \cos \left(\frac{3\pi}{4} + \frac{\pi}{8} \right) \right) = \frac{\cos \frac{\pi}{8} - \cos \left(\frac{3\pi}{4} + \frac{\pi}{8} \right)}{2 \sin \frac{\pi}{8}}$$

$$f(x) > \frac{2}{\pi} x \quad x \in (0, \frac{\pi}{2}); \quad y(x) = \frac{\sin x}{x}, \quad y' = \frac{x \cos x - \sin x}{x^2} = \frac{\cos x}{x^2} (x - \tan x) < 0$$

$$f(x) > x$$

$$f(x) = \tan x$$

Р.х.
 тапкан.

$$\frac{\sin x}{x} \Big|_{\frac{\pi}{2}} = \frac{2}{\pi}$$

$$\sum_{n=2}^{\infty} \frac{\cos \frac{\pi n}{4}}{n} \cdot \sin \frac{\pi n}{4} > \sum_{n=2}^{\infty} \frac{\ln^{100} n}{n} \sin \frac{\pi n}{4} = \sum_{n=2}^{\infty} \frac{\ln^{100} n}{n} \cdot \frac{1 - \cos \frac{\pi n}{2}}{2} = +\infty$$

3.1/p.

2.679

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1} \quad \text{по п. Лебана (нес.)}$$

2.680

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{(n+(-1)^n)^p} = \sum_{n=2}^{\infty} \frac{(-1)^n}{n^p (1 + \frac{(-1)^n}{n})^p}; \quad \frac{(-1)^n}{(n+(-1)^n)^p} = \frac{(n-(-1)^n)^p (-1)^n}{(n^2+1)^p}$$

$$\frac{1}{(2+1)^p} - \frac{1}{(2+1)^p} + \frac{1}{(4+1)^p} - \frac{1}{(4+1)^p} + \dots - \frac{(-1)^n}{(n+(-1)^n)^p} = \frac{(-1)^n}{n^p} \left(1 - \frac{p(-1)^n}{n} + O\left(\frac{1}{n^2}\right) \right) =$$

$$= \frac{(-1)^n}{n^p} - \frac{p}{n^{p+1}} + O\left(\frac{1}{n^{p+2}}\right); \quad 0 < p \leq 1 : \text{нес. гармон.}$$

$$\frac{1}{(n+1)^p} \leq \frac{1}{(n+1)^p} \leq \frac{1}{n^p} \quad p > 1 : \frac{(-1)^n}{(n+(-1)^n)^p} = O\left(\frac{1}{n^p}\right) = \text{нес. по п. Лебана}$$

$p \leq 0$: расх.
 $p = 1$: расх. адзон.

2.683

$$\sum_{n=1}^{\infty} (-1)^n \frac{n-1}{n+1} \cdot \frac{1}{n}, \quad a_n = (-1)^n \frac{n-1}{n+1} \cdot \frac{1}{n} = (-1)^n \frac{1}{n} \left(1 + O\left(\frac{1}{n}\right) \right) = \frac{(-1)^n}{n} + O\left(\frac{1}{n^2}\right)$$

2.684

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^{100}}{2^n} \quad \text{по п. Даламбера} \quad |a_n| = \frac{n^{100}}{2^n} \quad \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{(n+1)^{100}}{2^{n+1}} = \lim_{n \rightarrow \infty} \frac{1}{2} \left(1 + \frac{1}{n} \right)^{100} = \frac{1}{2} < 1$$

$$2 \left| \sum_{n=1}^{\infty} u_n \right| \leq C$$

$$\sum_{n=2}^{\infty} \frac{\sin \frac{\pi n}{12}}{\ln n} = \frac{1}{2 \sin \frac{\pi}{24}} \sum_{n=2}^{\infty} \frac{2 \sin \frac{\pi n}{12} \sin \frac{\pi}{24}}{\ln n} \quad \text{2.676}$$

$$i \sqrt{\frac{1}{n^2} \ln n} \frac{\cos \frac{\pi}{24} \cos \frac{\pi(n+1)}{24}}{2 \sin \frac{\pi}{24}} \leq \left| \frac{1}{\sin \frac{\pi}{24}} \right|$$

$$\cos \left(\frac{\pi n}{12} - \frac{\pi}{24} \right) \cos \left(\frac{\pi(n+1)}{12} + \frac{\pi}{24} \right) = \frac{\pi(n-\frac{1}{2})}{12} \frac{\pi(n+\frac{1}{2})}{12}$$

$$\left| \frac{\sin \frac{n\pi}{12}}{n} \right| \geq \frac{\sin \frac{2\pi}{12}}{n} = \frac{1 - \cos \frac{\pi}{6}}{2n} = \frac{1}{2n} - \frac{\cos \frac{\pi}{6}}{2n} ; \quad \sum_{n=2}^{\infty} \frac{1}{n \ln n} /$$

Прогр. сходимость

22699(66)

$$\sum_{n=2}^{\infty} \frac{(-1)^n \pi}{\ln n} = \sum_{n=2}^{\infty} \frac{(-1)^n \pi^{\frac{1}{2}}}{\ln n} ; \quad a_n = \frac{(-1)^n \cdot \pi^{\frac{1}{2}}}{n \cdot \frac{1}{n \ln n}} \dots$$

Кн. расб

Прогр. сч. знакочеред. рядов

$$\sum_{n=1}^{\infty} (-1)^{n-1} u_n, \quad \lim u_n = 0 \Rightarrow \text{сх.} ; \quad \left| \frac{u_{n+1}}{u_n} \right| \leq q < 1 - \text{сх. абсолютно}$$

$u_{n+1} \leq u_n$ $\geq 1 - \text{абс. не сх.}$

$$\sum_{n=1}^{\infty} u_n v_n, \quad u_n - \text{норм. сч. ун.}, \quad u \rightarrow 0 \quad \left. \begin{array}{l} \text{I пр. Абсче} \\ \text{сч. ун.} \end{array} \right\}$$

$\left| \sum_{k=1}^n u_k \right| \leq M$

$$\sum_{n=1}^{\infty} u_n v_n, \quad u_n - \text{норм. сч. ун.}, \quad v_n - \text{сч.} \quad \left. \begin{array}{l} \text{I пр. Абсче} \\ \text{сч.} \end{array} \right\}$$

$$u_n \downarrow 0, \quad a \left| \sum_{n=1}^k v_n \right| \leq M \Rightarrow \text{сх.}$$

норм. сч. ун.

$$\sum_{k=1}^n \sin kx = \frac{\cos \frac{x}{2} - \cos \left(\frac{n+1}{2} x \right)}{2 \sin \frac{x}{2}} ; \quad \sin \frac{x}{2} \neq 0 ; \quad \frac{x}{2} \neq \pi n, \quad n \in \mathbb{Z}$$

$x \neq 2\pi n, \quad n \in \mathbb{Z}$

$$\{ \sin x \} \in (0, 2\pi) \Rightarrow \sin x \geq 2x$$

$$\text{Если } \sin x \text{ убывает, то } \sum \sin x$$

$$\sum_{n=1}^n \cos nx = \frac{\cos \left(\frac{n+1}{2} x \right) - \cos \frac{x}{2}}{2 \sin \frac{x}{2}} \leq \frac{2}{2 \sin \frac{x}{2}} \leq \frac{2}{\pi x} \quad x \in (0, 2\pi)$$

$$\sum_{n=1}^{\infty} \frac{\cos nx}{n^{\alpha}} \quad \alpha > 0 \quad u_n = \frac{1}{n^{\alpha}} \downarrow 0, \quad \sum_{n=1}^{\infty} \cos nx \text{ сч. ун.}$$

$\sum_{n=1}^{\infty} \frac{|\cos nx|}{n^{\alpha}} \geq \sum_{n=1}^{\infty} \cos^2 nx$

Хардн Т.
Синг. релл
Панеж. релл
Панеж. релл

2682

$$\sum_{n=1}^{\infty} \frac{\sin \frac{\pi n}{4}}{n^p}$$

ASC. ex. : $\sum |u_n| \leq \sum \frac{1}{n^{p-1}}$ unv. q. Koun - Mann
ex. $p > 1$

ASC. cr. : $p > 2$

$p > 2$; $0 < p \leq 1$

$$\sum_{n=1}^{\infty} \frac{\sin \frac{\pi n}{4}}{n^p} \left(\frac{1}{1 + \frac{\sin \pi n}{4}} \right) ; \frac{1}{1+x} = 1 - x + x^2 - \dots$$

$$\sum_{n=1}^{\infty} \frac{\sin \frac{\pi n}{4}}{n^p} \left(1 - \frac{\sin \pi n}{4n^p} + O\left(\frac{1}{n^{2p}}\right) \right) = \sum_{n=1}^{\infty} \frac{\sin \frac{\pi n}{4}}{n^p} - \sum_{n=1}^{\infty} \frac{\sin^2 \frac{\pi n}{4}}{n^{2p}} = 1 - x + O(x)$$

ex. gen. $\frac{1}{2} \leq p \leq 1$

ASC. u gen. cr. pgen.

2698(b)

$$\sum_{n=10}^{\infty} \frac{\sin n}{n + 10 \sin n} ; \left| \frac{\sin n}{n + 10 \sin n} \right| \geq \left| \frac{\sin n}{n + 10} \right| \geq \left| \frac{\sin n}{n + 10} \right| \geq \frac{\sin^2 n}{n + 10} = \frac{1 - \cos 2n}{2(n + 10)}$$

$$\sum_{n=10}^{\infty} \frac{\sin n}{n} \left(\frac{1}{1 + \frac{10 \sin n}{n}} \right) = \sum_{n=10}^{\infty} \frac{\sin n}{n} \left(1 - \frac{10 \sin n}{n} + O\left(\frac{1}{n^2}\right) \right)$$

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n} + (-1)^n} = \sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n} \left(1 + \frac{(-1)^n}{\sqrt{n}} \right)} = \sum_{n=2}^{\infty} \frac{(-1)^n}{n^{\frac{1}{2}}} \left(1 - \frac{(-1)^n}{\sqrt{n}} + O\left(\frac{1}{n}\right) \right) = \sum_{n=2}^{\infty} \frac{(-1)^n}{n^{\frac{1}{2}}} - \sum_{n=2}^{\infty} \frac{1}{n} + O\left(\frac{1}{n}\right) = \text{pex.}$$

2680

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(n + (-1)^n)^p} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^p \left(1 + \frac{(-1)^n}{n} \right)^p} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^p} \left(1 - \frac{p(-1)^n}{n} + O\left(\frac{1}{n^2}\right) \right) =$$

2687

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(\sqrt{n} + (-1)^n)^p} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\frac{p}{2}} \left(1 + \frac{(-1)^n}{\sqrt{n}} \right)^p} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\frac{p}{2}}} \left(1 - \frac{p(-1)^n}{\sqrt{n}} + O\left(\frac{1}{n}\right) \right) =$$

2686
2690

neudk. $p=1$,
 $p > 0, p > 1$
 $p > 1$

2684

3 5 4 1 5 5 2 6
7 6 3 4 5 6 7 8
znano znano znano znano znano znano znano znano

$0 < p \leq 1$
ex. gen. no Neudk.

3,1415926535

2676

$$\sum_{n=2}^{\infty} \frac{\sin \frac{n\pi}{12}}{\ln n} = \sum_{n=2}^{\infty} \frac{\cos \frac{\pi}{24} - \cos(n+\frac{1}{2})\frac{\pi}{12}}{\sin \frac{\pi}{24}} \cdot \frac{1}{\ln n} \quad \text{Ap. Dir. - Abne part.}$$

$$\sum_{n=2}^{\infty} \frac{|\sin \frac{n\pi}{12}|}{\ln n} \geq \sum_{n=2}^{\infty} \frac{\sin^2 \frac{n\pi}{12}}{\ln n} =$$

$$= \sum_{n=2}^{\infty} \left(\frac{1}{2\ln n} - \frac{\cos \frac{n\pi}{6}}{2\ln n} \right) > +\infty;$$

$$|\sum| < \frac{1}{\sin \frac{\pi}{24}}$$

2677

$$\sum_{n=1}^{\infty} \frac{\sin n \sin n^2}{n} = \sum_{n=1}^{\infty} \frac{1}{2n} (\cos(n(n+1)) - \cos(n(n-1)))$$

$$\ominus \sum_{n=1}^{\infty} \frac{1}{2n} (\cos 0 - \cos(n(n+1))) \quad \text{cr. no Dir. - Abne.}$$

$n=1: \cos 0 - \cos 2 + \cos 0 - \cos 6 + \dots$

Abc. cr.: $\sum_{n=1}^{\infty} \left| \frac{\sin n \sin n^2}{n} \right| \geq \sum_{n=1}^{\infty} \frac{\sin^2 n \sin^2 n^2}{n} = \sum_{n=1}^{\infty} \frac{(1 - \cos 2n)(1 - \cos 2n^2)}{4n} =$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{4n} - \frac{\cos 2n}{4n} - \frac{\cos 2n^2}{4n} + \frac{\cos 2n \cos 2n^2}{4n} \right) \text{ part.}$$

2660

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \dots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k} = S, \quad S_{2n} =$$

$$S_{3n} = \frac{5}{4} + \frac{5}{32} + \dots + \frac{5}{2^{3n+5}} = \frac{5}{2^{3n+5}} \left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \dots + \frac{1}{2^{3n-3}} \right) + \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2^4} + \dots + \frac{1}{2^{3n-2}} \right)$$

$$- \left(\frac{1}{2^2} + \frac{1}{2^5} + \dots + \frac{1}{2^{3n-1}} \right) = \frac{1 - \frac{1}{2^{3n}}}{1 - \frac{1}{2^3}} + \frac{1}{2} \cdot \frac{1 - \frac{1}{2^{3n}}}{1 - \frac{1}{2^3}} - \frac{1}{2} \cdot \frac{1 - \frac{1}{2^{3n}}}{1 - \frac{1}{2^3}} = \left(1 + \frac{1}{2} - \frac{1}{2^2} \right) \frac{1 - \frac{1}{2^{3n}}}{1 - \frac{1}{2^3}} = \frac{5}{4} \cdot \frac{1 - \frac{1}{2^{3n}}}{1 - \frac{1}{2^3}}$$

$$S = \lim_{n \rightarrow \infty} S_{3n} = \frac{5}{4} \cdot \frac{1}{1 - \frac{1}{2^3}} = \frac{10}{7}.$$

2661

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{n} \quad \text{cr. no n. det. Summ.}$$

$$S_{2n} = \frac{1}{2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \dots + \frac{1}{(2n-1)(2n)} = \left(1 + \frac{1}{2} + \dots + \frac{1}{2n-1} \right) - 2 \left(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2n} \right) =$$

$$= C + \ln 2n + O_n - 2 \cdot \frac{1}{2} (C + \ln n + O_n) = \ln 2 + O\left(\frac{1}{n}\right).$$

2671

2672

1/1

$$\sum_{n=2}^{\infty} \ln(1 + \frac{(-1)^n}{n^p}) ; p > 0 : \ln(1 + \frac{(-1)^n}{n^p}) = \frac{(-1)^n}{n^p} - \frac{1}{2n^{2p}} + O(\frac{1}{n^{3p}})$$

(1), (2), (3) см. при $p > 1$

$\frac{1}{2} < p \leq 1$ (2), (3) не в

(1) см. по нр. $\ln(1 + \frac{(-1)^n}{n^p})$ $p > 0$; см. при $p > \frac{1}{2}$;

$$\frac{1}{2n^p} < |\ln(1 + \frac{(-1)^n}{n^p})| < \frac{2}{n^p} \Rightarrow \text{см. ас. при } p > 1 \Rightarrow \frac{1}{2} < p \leq 1 \text{ см. ун.}$$

§

$$\sum_{k=0}^{\infty} a_k \sum_{k=0}^{\infty} b_k = \sum_{k=0}^{\infty} c_k, \quad c_k = \sum_{i=0}^k a_i b_{k-i}$$

Кн. работа

см. метод
Линейного
Метода
Параллельного
Перемножения

Ас. см. теорема:

$$\sum_{k=0}^{\infty} a_k \sum_{k=0}^{\infty} b_k \text{ можно переписать без разницы как } \sum_{k=0}^{\infty} c_k \rightarrow AB$$

Теорема Если $A \in \mathbb{R}^{\infty}$, то при конв. см. в AB .
(теорема) $B \in \mathbb{R}^{\infty}$ $\sum |b_k| < \infty$

$$\sum_{n=1}^{\infty} a_n \pm \sum_{n=1}^{\infty} b_n \quad a_n, b_n - \text{см.} \\ = \sum_{n=1}^{\infty} (a_n \pm b_n)$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{3^n} \pm \frac{(-1)^n}{n^3} \right) + \sum_{n=1}^{\infty} \left(\frac{1}{3^{n+1}} \mp \frac{(-1)^{n+1}}{n^3} \right) = \frac{4}{3} \sum_{n=1}^{\infty} \frac{1}{3^{n+1}} = \frac{4}{3} \cdot \frac{1}{3} = \frac{4}{9}$$

$$1 - q = \frac{(1 - q^{n+1})}{(1 - q)}$$

$$q < 0, n \rightarrow \infty \Rightarrow \frac{1}{1 - q}$$

Σ 2709

$$\sum_{n=0}^{\infty} \cos\left(\frac{2\pi n}{3}\right)$$

$$= \sum_{k=0}^{\infty} \frac{1}{2^{3k}} + \sum_{k=0}^{\infty} -\frac{1}{2} \frac{1}{2^{3k+1}} + \sum_{k=0}^{\infty} -\frac{1}{2} \frac{1}{2^{3k+2}} =$$

$$\cos\left(\frac{2\pi n}{3}\right) = \begin{cases} 1, & n=3k \\ -\frac{1}{2}, & n=3k+1 \\ -\frac{1}{2}, & n=3k+2 \end{cases}$$

$$= \frac{1}{1 - \frac{1}{8}} - \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{8}} - \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{8}} = \frac{5}{7}$$

Σ 2700

$$\sum_{n=0}^{\infty} x^{nk} y^{\frac{n-1}{2}}$$

нужно

$$A_1 = \{n=2k\} \\ A_2 = \{n=2k+1\}$$

$$\sum_{n \in A} = \sum_{n \in A_1} + \sum_{n \in A_2} =$$

$$= \sum_{k=0}^{\infty} x^{2k} y^{\frac{2k-1}{2}} + y \sum_{k=0}^{\infty} x^{2k} y^k = (1+y) \sum_{k=0}^{\infty} (xy)^k = (1+y) \frac{1}{1-xy}$$

[xy] - наименьшее целое, не превыш. x

$\sqrt{2711}$
 $\sum_{n=0}^{\infty} \frac{1}{n!} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \sum_{n=0}^{\infty} C_n$, $C_n = \sum_{i=0}^n a_i b_{n-i}$, $a_i = \frac{1}{i!}$
 $n! C_n = \sum_{i=0}^n \frac{1}{i!} \frac{(-1)^{n-i}}{(n-i)!} = \sum_{i=0}^n C_n^i (-1)^{n-i} = (1-1)^n = 0 \Rightarrow \sum_{n=0}^{\infty} \frac{1}{n!} = e$
 $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$
 $b_n = \frac{(-1)^n}{n!}$

$\sqrt{2712}$
 $\sum_{n=0}^{\infty} q^n \sum_{n=0}^{\infty} q^n = \sum_{n=0}^{\infty} C_n$, $C_n = \sum_{i=0}^n q_i q_{n-i} = q^n \cdot (n+1)$
 $\sum_{n=0}^{\infty} C_n = \sum_{n=0}^{\infty} (n+1) q^n = \sum_{k=0}^{\infty} a_k x^k$ (generating function)
 $(\sum_{k=0}^{\infty} a_k x^k)' = \sum_{k=0}^{\infty} k a_k x^{k-1} \Big|_{x=1} = \sum_{k=1}^{\infty} a_k \cdot k$

$\sqrt{2713}$
 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} = \sum_{n=1}^{\infty} C_n$, $C_n = \sum_{i=1}^n \frac{(-1)^{i+1}}{\sqrt{i}} \cdot \frac{(-1)^{n-i+1}}{\sqrt{n-i+1}} = \sum_{i=1}^n \frac{1}{\sqrt{i} \sqrt{n-i+1}}$
 $\frac{a+b}{2} \geq \sqrt{ab}$, $a=i$, $b=n-i+1$; $\sqrt{i} \sqrt{n-i+1} \leq \frac{n+1}{2}$
 $\Rightarrow \frac{1}{\sqrt{i} \sqrt{n-i+1}} \geq \frac{2}{n+1}$
 $\Rightarrow$ pos. pos. as $n \rightarrow \infty$.

$\sqrt{2706}$
 a) ex: $\sum \frac{1}{n}$; $\sum \frac{(-1)^n}{n}$; $\sum \frac{1-q^n}{1-q}$
 pos.: $\sum 0$; $\sum \frac{1}{n}$
 b) ex: $\sum \frac{1}{n}$, $\sum -\frac{1}{n}$; pos.: $\sum \frac{1}{n}$, $\sum -\frac{1}{n}$

$\sqrt{2714}$
 $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^\alpha} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^\beta} = \sum_{n=1}^{\infty} C_n$, $C_n = \sum_{k=1}^n \frac{(-1)^{k-1}}{k^\alpha} \cdot \frac{(-1)^{n-k}}{(n-k+1)^\beta} = \sum_{k=1}^n \frac{(-1)^{n-1}}{k^\alpha (n-k+1)^\beta}$
 $b_n = \frac{(-1)^{n-1}}{n^\beta}$; $C_n \geq \sum_{k=1}^n \frac{1}{k^\alpha (n-k+1)^\beta} \geq \frac{1}{\left(\frac{n}{2}\right)^\alpha} \sum_{k=1}^n \frac{1}{(n-k+1)^\beta} \geq \frac{1}{\left(\frac{n}{2}\right)^\alpha} \sum_{k=2}^n \frac{1}{k^\beta} \geq \frac{1}{\left(\frac{n}{2}\right)^\alpha} \int_2^n \frac{dx}{x^\beta} = 2^\alpha \cdot \frac{1}{1-\beta} \left(1 - \frac{1}{2^{1-\beta}}\right)$
 $\Rightarrow \alpha + \beta < 1$, $C_n \rightarrow \infty$; $\alpha + \beta = 1 \Rightarrow C_n \geq 2^\alpha \cdot \frac{1}{1-\beta} \left(1 - \frac{1}{2^{1-\beta}}\right) > 0$, pos.

$\alpha + \beta > 1$: $C_n = \sum_{k=1}^n \frac{1}{k^\alpha (n-k+1)^\beta} + \sum_{k=2}^n \frac{1}{k^\alpha (n-k+1)^\beta} = \sum_1 + \sum_2$

$\frac{1}{k^\alpha (n-k+1)^\beta}$

2715

$$1 - \sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n, \quad 1 + \sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^{n-1} \left(2^n + \frac{1}{2^{n+1}}\right) \quad (\text{покажем, что } C_n \text{ имеет предел})$$

$$C_n = a_1 b_n + b_1 a_n + \sum_{k=2}^{n-1} a_k b_{n-k+1}, \quad a_1 = 1, a_n = \left(\frac{3}{2}\right)^{n-1}, b_1 = 1, b_n = \left(\frac{3}{2}\right)^{n-2} \left(2^{n-1} + \frac{1}{2^n}\right), n \geq 2$$

$$\Rightarrow C_n = \left(\frac{3}{2}\right)^{n-2} \left(2^{n-1} + \frac{1}{2^n}\right) - \left(\frac{3}{2}\right)^{n-1} - 4 \cdot 3^{n-2} \sum_{k=2}^{n-1} \frac{1}{2^k} - \frac{3^{n-2}}{2^{n-1}} \sum_{k=2}^{n-1} 2^k = \left(\frac{3}{4}\right)^{n-1}$$

$$\Rightarrow \sum_{n=1}^{\infty} C_n = \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^{n-1} = 4.$$

Бесконечные произведения

$$a_n > 0, p_n = a_1 a_2 \dots a_n, \quad \lim_{n \rightarrow \infty} p_n = p;$$

$$\prod_{k=1}^{\infty} a_k \text{ сходятся к } p: \quad 1) 0 \quad 2) \pm \infty$$

$$\text{Критерий Вейерштрасса: } \forall \varepsilon > 0 \exists n_0(\varepsilon) \forall n \geq n_0 \quad \left| \frac{p_{n+1}}{p_n} - 1 \right| < \varepsilon$$

$$a_{n+1} = 1 + u_n,$$

$$\sum \ln a_k = \sum \ln(1 + u_k)$$

$$\ln(1 + u_n) \sim u_n \quad \Rightarrow \quad \sum \ln(1 + u_k) \sim \sum u_k.$$

$$\sum_{k=1}^{\infty} \ln(1 + |u_k|) < \infty \quad \Leftrightarrow \quad \sum_{k=1}^{\infty} u_k < \infty$$

$$\sum u_n < \infty$$

$$\sum u_n^2 < \infty$$

$$\sum u_n \text{ сходятся к } p.$$

$$\prod_{k=2}^{\infty} \left(1 - \frac{1}{k^2}\right) = \frac{1}{2}, \quad p_n = \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \dots \left(1 - \frac{1}{(n-1)^2}\right) \left(1 - \frac{1}{n^2}\right) =$$

$$= \frac{2}{n^2} \cdot \frac{8}{3^2} \dots \frac{(n-1)^2 - 1}{(n-1)^2} \cdot \frac{n^2 - 1}{n^2} = \frac{(n-1)(n+1)}{(n-1)^2} \cdot \frac{(n-1)(n+1)}{n^2} = \frac{1}{2} + \frac{n+1}{n} \rightarrow \frac{1}{2}$$

$$\prod_{k=1}^{\infty} \left(1 + \frac{1}{k}\right) \sim \sum_{k=1}^{\infty} \ln\left(1 + \frac{1}{k}\right) \sim \sum_{k=1}^{\infty} \frac{1}{k},$$

$$\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right) \cdot \dots \cdot \left(1 + \frac{1}{n-1}\right)\left(1 + \frac{1}{n}\right) = n+1 \rightarrow \infty.$$

$$\prod_{k=1}^{\infty} \frac{e^{\frac{1}{k}}}{1 + \frac{1}{k}} = \frac{e^{1 + \frac{1}{2} + \dots + \frac{1}{n-1} + \frac{1}{n}}}{2\left(1 + \frac{1}{2}\right) \cdot \dots \cdot \left(1 + \frac{1}{n-1}\right)\left(1 + \frac{1}{n}\right)}, \quad 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} = \ln n + C + \varepsilon_n$$

$$p_n = \frac{e^{\ln n + C + \varepsilon_n}}{n+1} = \frac{n}{n+1} \cdot e^{C + \varepsilon_n} \xrightarrow{n \rightarrow \infty} e^C$$

Зад 55

$$\prod_{k=1}^{\infty} \cos\left(\frac{\pi}{2^{k+1}}\right) \stackrel{?}{=} \frac{1}{2}, \quad p_n = \cos\left(\frac{\pi}{2^2}\right) \cdot \cos\left(\frac{\pi}{2^3}\right) \cdot \cos\left(\frac{\pi}{2^4}\right) \cdot \dots \cdot \cos\left(\frac{\pi}{2^{n+1}}\right) \cdot \frac{1 \sin \frac{\pi}{2}}{2 \sin \frac{\pi}{2^{n+1}}} = \frac{\sin \frac{\pi}{2}}{2^n \sin \frac{\pi}{2^n}} = \frac{1}{2}$$

Зад 68

$\prod_{k=1}^{\infty} \left(1 + \frac{1}{k^p}\right)$ сходящееся с.с. кб с.с. $p > 1$; $p \leq 1$ д.с.с.

$$p_n = \left(1 + \frac{1}{1^p}\right)\left(1 + \frac{1}{2^p}\right) \cdot \dots \cdot \left(1 + \frac{1}{(n-2)^p}\right)\left(1 + \frac{1}{(n-1)^p}\right)\left(1 + \frac{1}{n^p}\right) = \frac{2}{1} \cdot \frac{5}{2^2} \cdot \frac{10}{3^2} \cdot \dots$$

$$\frac{n^2 - 2n + 3}{(n-2)^2} \cdot \frac{n^2 - 2n + 2}{(n-1)^2} \cdot \frac{n^2 + 1}{n^2} = \dots \quad p > 1 \text{ с.с.}, \text{ при } p \leq 1 \text{ д.с.с.}$$

$\prod_{k=1}^{\infty} \left(1 + \frac{1}{k(k+2)}\right)$ с.с. н.с. с.с. с.с. ;

$$p_n = \left(1 + \frac{1}{1 \cdot 3}\right)\left(1 + \frac{1}{2 \cdot 4}\right) \cdot \dots \cdot \left(1 + \frac{1}{(n-2)n}\right)\left(1 + \frac{1}{(n-1)(n+1)}\right)\left(1 + \frac{1}{n(n+2)}\right) =$$

$$= \frac{n^2 - 2n + 1}{(n-2)n} \cdot \frac{n^2}{(n-1)(n+1)} \cdot \frac{n^2 + 2n + 1}{n(n+2)} = \frac{n+1}{n+2} = 1 - \frac{1}{n+2} \rightarrow 1.$$

Зад 71

$$\prod_{k=1}^{\infty} \frac{k^2 + a_1 k + b_1}{k^2 + a_2 k + b_2} = \prod_{k=1}^{\infty} \left(1 + \frac{(a_1 - a_2)k + b_1 - b_2}{k^2 + a_2 k + b_2}\right), \text{ если } a_1 \neq a_2 \Rightarrow \text{д.с.с. н.с. с.с.}$$

$$a_1 = a_2 \Rightarrow \text{с.с.}$$

Зад 90

$$\sum_{k=1}^{\infty} \prod_{r=1}^{\infty} \left(1 + \frac{(-1)^{k+1}}{k^r}\right) \quad p > 1 \text{ с.с. д.с.с.},$$

$$\frac{1}{2} < p \leq 1 \text{ с.с. д.с.с.},$$

$$p \leq \frac{1}{2} \text{ д.с.с.}$$

$$\sum_{k=1}^{\infty} \left(\frac{(-1)^{k+1}}{k^p} - \frac{1}{2^p} + O\left(\frac{1}{k^{2p}}\right) \right)$$

2/4
3090
3091

$$\prod_{n=1}^{\infty} \cos \frac{x}{2^n} = \cos \frac{x}{2} \cdot \frac{2 \cos \frac{x}{2^n} \sin \frac{x}{2^n}}{2 \sin \frac{x}{2^n}} = \frac{1}{2^n \sin \frac{x}{2^n}} \sin x = \frac{x/2^n}{\sin x/2^n} \cdot \frac{\sin x}{x} \rightarrow \frac{\sin x}{x}$$

3096

$$\prod_{n=1}^{\infty} \frac{a_n}{b_n}; \quad \left| \frac{p_n}{p_{n+1}} - 1 \right| = \left| \frac{p_n}{p_{n+1}} \right| \cdot \left| \frac{p_{n+1}}{p_n} - 1 \right| = 1; \quad \ln a_n \sim \sum_{n=1}^{\infty} 2 \frac{\ln n}{n} \quad \text{no m.p.}$$

3092

$$\prod_{n=1}^{\infty} \left(1 + \frac{x}{n}\right) e^{-\frac{x}{n}} \quad \ln a_n = \ln \left(1 + \frac{x}{n}\right) \sim \frac{x}{n} \Rightarrow \text{pas-c.}$$

3091

$$\prod_{n=2}^{\infty} \left(1 + \frac{(-1)^n}{\ln n}\right) \quad \text{pas-c.} \quad \sum_{n=2}^{\infty} \frac{1}{\ln^2 n} \quad \text{A.E. ex. 1 pub. a.E. ca.}$$

$$E_n = n \left(1 - \left(\frac{\ln n}{\ln n+1}\right)^2\right) = n \frac{\ln^2 n+1 - \ln^2 n}{\ln^2 n+1} = n \frac{(\ln n+1)(\ln n-1)}{\ln^2 n+1} =$$

$$= n \frac{\ln n(\ln n)}{\ln^2 n+1} \left(\frac{1}{n} + O\left(\frac{1}{n}\right)\right) = \frac{\ln n(\ln n)}{\ln^2 n+1} (1 + O(1)) = \frac{\ln^2 n (1 + \frac{1}{n})}{\ln^2 n (1 + \frac{1}{n})} (1 + O(1));$$

$$n > \ln^2 n, \quad \frac{1}{\ln^2 n} > \frac{1}{n} \Rightarrow \text{up. cp. pas-c.}$$

3092

$$\prod_{n=2}^{\infty} \frac{\ln n}{\sqrt{n} + (-1)^n} \quad p_n = \frac{\sqrt{n}(\sqrt{n} + (-1)^{n+1})}{n-1}; \quad \ln p_n = \frac{1}{2} \ln n - \ln(\sqrt{n} + (-1)^n)$$

$$\ln p_n = \ln \left(1 - \frac{(-1)^n}{\sqrt{n} + (-1)^n}\right), \quad u_k = \ln p_k + \ln p_{k+1}$$

$$u_k = \ln \left(1 - \frac{1}{\sqrt{2k+1}}\right) + \ln \left(1 + \frac{1}{\sqrt{2k+1}+1}\right) =$$

$$= \ln \left(1 - \frac{\sqrt{2k+1} - \sqrt{2k} - 1}{(\sqrt{2k}+1)(\sqrt{2k+1}-1)}\right) > 0;$$

$$u_k = -\frac{\sqrt{2k+1} - \sqrt{2k} - 1}{(\sqrt{2k}+1)(\sqrt{2k+1}-1)} + O\left(\frac{k^{1/2}}{k}\right) \quad u_k = O^*\left(\frac{1}{k^{1/2}}\right) \quad 1/2 < 1 \Rightarrow \text{pas-c.}$$

3091

$$\prod_{n=1}^{\infty} \frac{n^{(-1)^n}}{p_n} \quad \ln p_n = \ln n^{(-1)^n \cdot \frac{1}{n}} = \frac{(-1)^n}{n} \ln n \quad \text{no m.p. a.E. pas-c.}$$

$\{f_n(x)\}_{n=1}^{\infty}$
 $a \leq x \leq b$

$\lim_{n \rightarrow \infty} f_n(x) = f(x) \quad a < x < b$

сх. непрерывная функция
 не имеет разрывов

$D(f, (x)) = (a, b), \quad \forall \varepsilon > 0 \exists N(\varepsilon) \forall n \geq N \quad |f_n(x) - f(x)| < \varepsilon$

Дж. Т.

$$|f_n(x) - f(x)| < \epsilon, f_n(x) \xrightarrow[n \rightarrow \infty]{\text{to } C_3} f(x)$$

$\lim_{n \rightarrow \infty} \sum_{k=1}^n u_k(x)$

Если cx - nos. $\Rightarrow cx$ - nos.
 cx - pobit. $\Rightarrow cx$ - pobit.

$$\forall \varepsilon > 0 \exists N(\varepsilon) \forall n \geq N(\varepsilon) \forall x \in [a, b] |u_{n+p}(x) - u_n(x)| < \varepsilon$$

Теор $f_n(x)$ мон. и $f'_n(x)$ равн. сж. $\Rightarrow f'_n(x) \rightarrow f'(x)$
и $\{f_n(x)\}$ сж. тогда в
справ. т.

~~$$\text{Prop. } \frac{d}{dx} \left(\sum_{n=1}^{\infty} a_n(x) \right) =$$~~
$$\Rightarrow \forall \varepsilon > 0 \quad \exists N = N(\varepsilon) \quad \forall n \geq N, \forall x \in [a, b] \quad |f_n(x) - f(x)| < \varepsilon \quad \Rightarrow$$
$$\Rightarrow \sup_{a \leq x \leq b} |f_n(x) - f(x)| < \varepsilon \Rightarrow f_n \rightarrow 0$$

($\Leftarrow$) ~~geg~~ $\forall \epsilon > 0 \quad \exists N = N(\epsilon) : \forall n \geq N \quad \sup_{x \in [a,b]} |f_n(x) - f(x)| < \epsilon \Rightarrow$ ist convergent
 $\forall x \in [a,b]$

Средна верес. ил. ~~скривн.~~ ск.

1) Найти отпр. всех f_n ; $D(f_n(x)) = ?$

2) $f'(x) = \lim_{n \rightarrow \infty} f'_n(x)$ по тожд. пр.з. на $[a, b]$

$$3) \Gamma_n(x) = |f_n(x) - f(x)|$$

$$4) \Gamma'_n(x) = 0, \text{ т. экстр.}, \quad x_k - \text{т. экстр.} \text{ экстр. } \Gamma_n(x_k) = \max_{x \in [0,1]} \Gamma_n(x)$$

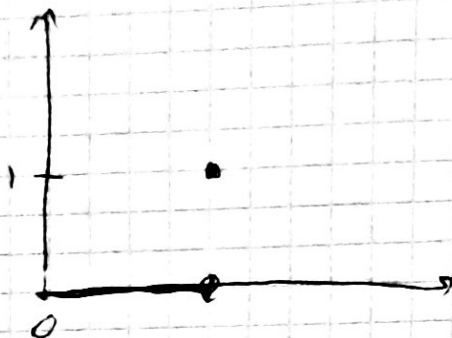
$$5) \lim_{n \rightarrow \infty} \Gamma_n(x_k) = \begin{cases} 0 - \text{равн. экстр.} & \text{не 0 экстр. равн.} \\ \text{не 0 экстр. равн.} & \text{не 0 экстр. равн.} \end{cases}$$

$f_n(x) = x^n, x \in [0,1];$ монотонно убывает на $[0,1]$.

$$1) D(f_n(x)) = [0,1]$$

$$2) \lim_{n \rightarrow \infty} x^n = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & x = 1 \end{cases}$$

при $x \in [0,1]$ монотонно убывает.



$$\forall \epsilon > 0, \exists n_0 = \frac{\ln \epsilon}{\ln x}, \lim_{x \rightarrow 1-0} \frac{\ln \epsilon}{\ln x} = \infty \Rightarrow \exists n = n(\epsilon) \Rightarrow \begin{cases} 0 \leq x < 1 \text{ по } \epsilon \\ 0 \leq x < 1 \text{ по } \epsilon \end{cases}$$

$$x^n \leq (1-\delta)^n \leq \epsilon \Rightarrow \ln x = \ln(1-\delta)$$

$$n \geq \frac{\ln(1-\delta)}{\ln x}$$

$$\ln(1-\delta) < \ln \epsilon$$

$$n_0 = \left\lceil \frac{\ln \epsilon}{\ln(1-\delta)} \right\rceil + 1 \quad \forall n \geq n_0$$

2746

$$f_n(x) = x^n, \quad 0 \leq x \leq \frac{1}{2} \quad x^n \leq \left(\frac{1}{2}\right)^n < \epsilon, \quad \frac{1}{2} \geq \ln \epsilon, \quad \text{...}$$

2747

$$f_n(x) = x^n - x^{n+1}, \quad 0 \leq x \leq 1; \quad \lim_{n \rightarrow \infty} x_n \rightarrow 0, \quad 0 \leq x < 1.$$

$$f(x) \equiv 0, \quad 0 \leq x \leq 1$$

$$x_n \rightarrow 1, \quad x = 1$$

$$\Gamma_n(x) = x^n - x^{n+1} = x^n(1-x);$$

$$\Gamma'_n(x) = nx^{n-1}(1-x) - x^n = x^{n-1}(n(1-x) - x) = 0 \Rightarrow x = \frac{n}{n+1} \text{ т. экстр. } f_n(x)$$

$$Z_n(x) = \left(\frac{n}{n+1}\right)^n \left(1 - \frac{n}{n+1}\right) = \frac{n^n}{(n+1)^{n+1}} = \frac{1}{n+1} \left(\frac{n}{n+1}\right)^n = \frac{1}{n+1} \left(1 + \frac{-1}{n+1}\right)^n \rightarrow 0$$

$\Rightarrow$ равн. экстр.

2748

$$f_n(x) = x^n - x^{2n}, \quad 0 \leq x \leq 1 \text{ мон. на } [0,1].$$

$$D(f_n) = [0, 1]; \text{ де } \text{max}, 0 \leq x < 1; x=1 \quad f_n'(1) = 0.$$

$$f_n(x) \equiv 0, 0 \leq x < 1, \quad \Gamma_n(x) = x^n - x^{2n}, \quad \Gamma_n'(x) = nx^{n-1} - 2n \cdot x^{2n-1} = x^{n-1}(n-2x^2) \\ \Rightarrow nx^{n-1}(1-2x^2) = 0 \Rightarrow x = \frac{1}{\sqrt{2}} \text{ - max.}$$

$$\Gamma_n\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} - \frac{1}{2} = \frac{1}{4} \Rightarrow \lim_{n \rightarrow \infty} \Gamma_n = \frac{1}{4} \text{ не с. публ.}$$

$$\forall \varepsilon > 0 \exists N(\varepsilon) \forall n \geq N \left| f_n(x) - f(x) \right| < \varepsilon \text{ - uniform conv.} \\ \exists \varepsilon > 0 \forall N(\varepsilon) \exists n \geq N \exists x \in [0, 1] \left| f_n(x) - f(x) \right| \geq \varepsilon.$$

2743

$$f_n(x) = \frac{1}{x+n}, x > 0; \quad 1) D(f) = (0, +\infty)$$

$$2) \lim_{n \rightarrow \infty} f_n(x) = 0 \Rightarrow f(x) = 0. \quad \Gamma_n(x) = \frac{1}{x+n} < \frac{1}{n} < \varepsilon, \quad n \geq \left\lceil \frac{1}{\varepsilon} \right\rceil + 1$$

2750

$$f_n(x) = \frac{nx}{1+n+x}, 0 \leq x \leq 1, \quad \lim_{n \rightarrow \infty} f_n(x) = \frac{x}{1+\frac{1+x}{n}} \equiv x = f(x) \text{ в.г.}$$

$$\Gamma_n(x) = \left| \frac{nx}{1+n+x} - x \right| = \left| \frac{nx - x - nx - x^2}{1+n+x} \right| \leq \frac{x(x+1)}{1+n} = \frac{x^2+x}{1+n} \leq \frac{2}{1+n} < \varepsilon \\ \Rightarrow n \geq \left\lceil \frac{2}{\varepsilon} \right\rceil + 1$$

$$\forall x \in [0, 1], \quad f_n \xrightarrow[n \rightarrow \infty]{} f(x).$$

2751

$$f_n(x) = \frac{2nx}{1+n^2x^2}, \quad 0 \leq x \leq 1$$

$$\lim_{n \rightarrow \infty} \frac{2nx}{1+n^2x^2} = 0 \quad \forall x \in [0, 1]. \quad \Gamma_n(x) = \left| \frac{2nx}{1+n^2x^2} \right|, 0 \leq x \leq 1$$

$$\Gamma_n'(x) = \frac{2n(1+n^2x^2) - 2nx(2n^2x)}{(1+n^2x^2)^2} = \frac{-2n^3x^2 + 2n}{(1+n^2x^2)^2}$$

$$= \frac{2n(1-n^2x^2)}{(1+n^2x^2)^2} = 0;$$

$$2n(1-n^2x^2) = 0, \quad 1-n^2x^2 = 0; \quad \Gamma_n(x_{\max}) = \Gamma_n\left(\frac{1}{n}\right) \rightarrow 1 \Rightarrow \text{не публ.}$$

277

277

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} \cdot \left(\frac{1-x}{1+x}\right)^n; \quad \lim_{n \rightarrow \infty} \sum_{n=1}^{\infty} u_n = ? \quad S_u = -1 \left(\frac{1-x}{1+x} \right) + \frac{1}{3}$$

$$\lim_{n \rightarrow \infty} \frac{\left| \frac{(-1)^n}{2n-1} \right|}{\left| \frac{(-1)^{n-1}}{2n-3} \right|} = 1, \quad \left| \frac{1-x}{1+x} \right| < 1, \quad (1-x)^2 < (1+x)^2, x > 0;$$

$$x < 0: \sum_{k=1}^{\infty} \frac{(-1)^k}{2k-1}, \quad x = 0: \left(\frac{1-x}{1+x}\right)^n \rightarrow 100.$$

ca. upu $x \leq 0$ u pux-gu upu $x < 0$.

2242

a) $\forall \varepsilon > 0 \exists N \in \mathbb{N} \forall n \geq N \forall x \in (x_0, +\infty) |f_n(x) - f(x)| < \varepsilon$

b) $\forall \varepsilon > 0 \exists N \in \mathbb{N}(\varepsilon, \varepsilon_0, b) : \forall n \geq N, \forall x \in (c, b) |f_n(x) - f(x)| < \varepsilon$

c) $\forall \varepsilon > 0 \exists N \in \mathbb{N}(\varepsilon) \forall n \geq N \forall x \in (x_0, +\infty) |f_n(x) - f(x)| < \varepsilon$

2251

$f_n(x) = \frac{x^n}{1+x^n}$, a) $0 \leq x \leq 1-\varepsilon$, b) $1-\varepsilon \leq x \leq 1+\varepsilon$, c) $1+\varepsilon \leq x < +\infty$, d) $x > 0$.

lim. na paku. ca; a) $\lim_{n \rightarrow \infty} \frac{x^n}{1+x^n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{x^n}\right)^{-1} = 0$

$$f_n(x) = \frac{x^n}{1+x^n}, \quad f'_n(x) = \frac{n x^{n-1} (1+x^n) - x^n (n x^{n-1})}{(1+x^n)^2} = \frac{n x^{n-1} (1-x^n)}{(1+x^n)^2} = 0 \Rightarrow x = 0$$

$$\frac{x^n}{1+x^n} < (1-\varepsilon)^n < \varepsilon' \Rightarrow n > \frac{\lg \varepsilon'}{\lg(1-\varepsilon)}, \quad N = \left\lfloor \frac{\lg \varepsilon'}{\lg(1-\varepsilon)} \right\rfloor + 1;$$

$$b) f(x) = \lim_{n \rightarrow \infty} f_n(x) = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & 1 \leq x < +\infty \end{cases}$$

$$0 < \varepsilon_0 < \frac{1}{3}, \quad x = \frac{1}{\sqrt[3]{\varepsilon_0}} : |f_n(x) - f(x)| = \frac{1}{3} > \varepsilon_0 \Rightarrow \text{ne paku. ca.}$$

$$c) \lim_{n \rightarrow \infty} f_n(x) = 1 = f(x), \quad |f_n(x) - f(x)| = \frac{1}{1+x^n} < \frac{1}{(1+\varepsilon)^n}, \quad \varepsilon' > 0,$$

$$\frac{1}{(1+\varepsilon)^n} < \varepsilon', \quad n > \frac{\lg \frac{1}{\varepsilon'}}{\lg(1+\varepsilon)}, \quad N = \left\lfloor \frac{\lg \frac{1}{\varepsilon'}}{\lg(1+\varepsilon)} \right\rfloor + 1 \quad \text{ca paku.}$$

2253

$$f_n(x) = \sqrt{x^2 + \frac{1}{n^2}}, \quad -\infty < x < +\infty$$

$$\lim_{n \rightarrow \infty} f_n(x) = |x|, \quad \left| |x| - \sqrt{x^2 + \frac{1}{n^2}} \right| = \frac{x^2 + \frac{1}{n^2} - x^2}{\sqrt{x^2 + \frac{1}{n^2}} + |x|} < \frac{\frac{1}{n^2}}{\frac{1}{n}} = \frac{1}{n} \Rightarrow N = \left\lceil \frac{1}{\varepsilon} \right\rceil$$

2273

$$\sum_{n=1}^{\infty} \frac{nx}{(1+x)(1+2x) \dots (1+nx)}$$

$$a) 0 \leq x \leq \varepsilon, \quad \varepsilon > 0$$

$$b) \varepsilon \leq x < +\infty$$

$$f_n(x) = \frac{nx}{(1+x)(1+2x) \dots (1+nx)}$$

$$a) \lim_{n \rightarrow \infty} f_n(x) = \lim_{n \rightarrow \infty} \frac{1}{(1+\frac{1}{n})(1+\frac{2}{n}) \dots (1+\frac{n}{n})} =$$

$$= \left(1 - \frac{1}{n} + o\left(\frac{1}{n}\right)\right) \left(1 - \frac{2}{n} + o\left(\frac{1}{n}\right)\right) \dots \left(1 - \frac{n-1}{n} + o\left(\frac{1}{n}\right)\right);$$

$$\lim_{n \rightarrow \infty} \frac{u_{n+1}(x)}{u_n(x)} = \lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \cdot \frac{1}{1+(n+1)x} \right) = 0 < 1,$$

$$S_1(x) = \sum_{k=1}^{\infty} u_k(x) = \sum_{k=1}^{\infty} \frac{kx}{(1+x) \cdot (1+kx)} = \sum_{k=1}^{\infty} \left(\frac{1}{(1+x) \cdot (1+(k-1)x)} - \frac{1}{(1+x) \cdot (1+kx)} \right) =$$

$$= 1 - \frac{1}{(1+x)(1+2x) \dots (1+nx)} \rightarrow 1 \quad \text{as } n \rightarrow \infty, \quad f(x) = \lim_{n \rightarrow \infty} S_n(x) = \begin{cases} 0, & x=0 \\ 1, & x>0 \end{cases}$$

a) $|g_n(x) - g(x)| = \frac{1}{(1+x)(1+2x) \cdots (1+nx)}$, $0 < \varepsilon_0 < 1$;

$$\lim_{x \rightarrow 0} \frac{1}{(1+x)(1+2x) \cdots (1+nx)} = 1, \quad \frac{1}{(1+x_0)(1+2x_0) \cdots (1+nx_0)} > \varepsilon_0,$$

$|f_n(x_0) - f(x_0)| > \varepsilon_0 \Rightarrow$ неравен. не не равн. не

$$\begin{aligned} \text{d) } x \geq \varepsilon, \quad n \geq 3: \quad |u_n(x)| &= \left| \frac{n x}{(1+x)(1+2x) \dots (1+nx)} \right| < \frac{n x}{(1+x)^n} = \frac{n x}{1+nx + \frac{1}{2!} n(n-1)x^2 + \dots + x^n} \\ &< \frac{n x}{\frac{1}{3!} n(n-1)(n-2)x^3} = \frac{6}{(n-1)(n-2)x^2} < \frac{6}{(n-2)\varepsilon^2} \quad \text{Für jedes } \sum_{i=1}^{\infty} \frac{6}{(n-2)\varepsilon^2} \quad \text{d.h. } \Rightarrow \text{Cauchy-Kriter.} \end{aligned}$$

Кн. работа

пр. Вернунг. $\sum_{k=1}^{\infty} |u_k| \leq \sum c_k < \infty \Rightarrow$ равном. сд. $\sum u_k$

Тогда $\left| \sum_{k=1}^n u_k(x) \right| \leq \sum_{k=1}^n |u_k(x)|$ Также газ. признак

$\Sigma(a_n \sim c_n)$ parv. $\Sigma(a_n)$ pers.
 — nris monap.

$$\sum_{k=1}^{\infty} u_k v_k \quad \text{ск. равн. на } x \in D$$

Един ω неизм. $\times$ равн. вентиль. в Директне

$$\sum u_k \text{ p.c.}, \quad u_k \text{ mon.} \Rightarrow \sum u_k v_k \text{ p.c.}$$

$$|v_k| \leq \epsilon \quad \forall k \in \mathbb{N}$$

by ~~Hand~~

усл-е восстановления равн. сил. X ;

Ap. Duperré - Acad

печ. вост. сун. рвбн. 4 ар.

м. Думе правн. см. роза

дл. 10^2 u_k нечет. и сумма чет. $\rightarrow$ u_k нечетно $\Rightarrow$ u_k равноч.

f_1 пер. пр. дес-факт пер., f_1^* мате. невоз. $\Rightarrow$ сн. равновесие

К. Коун пабыр. чк

2274

a) $\sum_{n=1}^{\infty} \frac{1}{x^2 + n^2} \leq \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$ no ~~Beispiel~~ - np.

b) $\sum_{n=1}^{\infty} \frac{x}{1+x^2 n^4} \leq \sum_{n=1}^{\infty} \frac{x}{2x n^4} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^4} < \infty \Rightarrow$ побн. чк.
 $x \in (0, \infty)$

2) $\sum_{n=1}^{\infty} \frac{n x}{1+n^5 x^2}$, $\left(\frac{n x}{1+n^5 x^2} \right)' = \frac{n(1+5x^2) - 2x n^5 \cdot n x}{(1+n^5 x^2)^2} = \frac{n - n^6 x^2}{(1+n^5 x^2)^2} < 0 \Rightarrow$

$\sum_{n=1}^{\infty} u_n(x) \Big|_{x=\frac{1}{n^5}} = \sum_{n=1}^{\infty} \frac{1}{1+n^5} \cdot \frac{1}{2} n^{-\frac{1}{2}} \Rightarrow$ no np. Коун чк. $\Rightarrow x = \sqrt{\frac{1}{n^5}}$

g) $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n!}} \left(x^n + \frac{1}{x^n} \right) \leq \sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n!}} (2^{n+1})$
 $\frac{1}{2} \leq |x| \leq 2$
 $\frac{u_{n+1}}{u_n} = \frac{(n+1)^2 \cdot 2^{n+2}}{\sqrt{(n+1)!}} \cdot \frac{\sqrt{n!}}{n^2 \cdot 2^{n+1}} =$

$\sum_{n=1}^{\infty} \frac{x^n}{\left[\frac{n}{2} \right]!}, \quad |x| < a$

$= \left(\frac{n+1}{n} \right)^2 \cdot 2 \cdot \frac{1}{\sqrt{n+1}} \rightarrow 0$

$\begin{cases} n=2k \\ n=2k+1 \end{cases} \leq \sum_{k=1}^{\infty} \frac{a^{2k}}{2k!} - \text{чк. no Daint.}$
 no ~~Beispiel~~.

$\Rightarrow$ no Daint.
Beispiel.

$n=2k+1$, аналогично

e) $\sum_{n=1}^{\infty} \frac{1}{n \sqrt{n}} \leq \sum_{n=1}^{\infty} \frac{1}{n \sqrt{n}} < \infty$ чк. побн.
 no unt. npym. Коун-Маринг.

k) $\sum_{n=2}^{\infty} \ln \left(1 + \frac{x^2}{n \ln^2 n} \right), \quad |x| < a$
 $\sum_{n=2}^{\infty} \ln \left(1 + \frac{x^2}{n \ln^2 n} \right) = \sum_{n=2}^{\infty} \left(\frac{x^2}{n \ln^2 n} + O \left(\frac{x^4}{n \ln^4 n} \right) \right) \leq \sum_{n=2}^{\infty} \frac{a^2}{n \ln^2 n} + O \left(\frac{1}{n \ln^2 n} \right)$
no Коун-Маринг
чк
no ~~Beispiel~~ чк

2275

$\sum_{n=1}^{\infty} \frac{\sin nx}{n}$, a) $\varepsilon \leq x \leq 2\pi - \varepsilon, \quad \varepsilon > 0$
 б) $0 \leq x \leq 2\pi$

$\sum_{k=1}^{\infty} \frac{\sin kx}{k}$ - нон. 2-сум. побн. чк.
оцен.

$\frac{1}{2 \sin \frac{x}{2}} \left(2 \sin \frac{x}{2} \cdot \sin x + 2 \sin \frac{x}{2} \sin 2x + \dots + 2 \sin \frac{x}{2} \sin nx \right) =$

$$= \left(\cos \frac{x}{2} + \cos \left(\frac{3x}{2} \right) + \cos \left(\frac{5x}{2} \right) + \dots - \cos \frac{x}{2} (n+1) \right) \frac{1}{2 \sin \frac{x}{2}}$$

$V_n = \frac{1}{n} \rightarrow 0$ монотон. убывает и ~~равн. с. к 0~~ ~~равн. с. к 0~~.
по пр. Абеля с. равномерно.

доказ. разн.

$\exists \varepsilon > 0 \forall N > 0 \exists n > N \exists p \in \mathbb{N}:$

$$\left| \sum_{k=1}^n \frac{\sin kx}{k} \right|_{x=\frac{1}{n}} = \left| \frac{\sin(n+1)x}{n+1} + \frac{\sin(n+2)x}{n+2} + \dots + \frac{\sin 2nx}{2n} \right|_{x=\frac{1}{n}} =$$

$$= \left| \frac{\sin \frac{n+1}{n}}{n+1} + \dots + \frac{\sin 2}{2n} \right| \geq \frac{\sin 1}{2} = \varepsilon_0 \text{ по лемме с. разн.}$$

$\sum_{n=1}^{\infty} \frac{\sin nx}{n}$ — с. разн. и равномерно;

$$\sum_{n=1}^{\infty} \left| \frac{\sin nx}{n} \right| \geq \sum_{n=1}^{\infty} \frac{\sin^2 nx}{n} = \sum_{k=1}^{\infty} \frac{1}{k} - \sum_{k=1}^{\infty} \frac{\cos 2kx}{2k} \Rightarrow \text{разн.}$$

872782

$\sum_{n=1}^{\infty} \varphi_n(x)$, φ_n монот. на $[a, b]$, $b \neq a$ и $b \neq \beta$ с.к.

$\Rightarrow$ разн. с. разн.

$$|\varphi_n(x)| \leq \max\{|\varphi_n(a)|, |\varphi_n(b)|\}, \quad \sum \max\{|\varphi_n(a)|, |\varphi_n(b)|\} < \infty$$

$$|R_n(x)| = \sum_{k=n+1}^{\infty} |\varphi_k(x)| \Rightarrow \forall \varepsilon > 0 \exists N(\varepsilon) : \forall n > N \quad \left| \sum_{k=n+1}^{\infty} \varphi_k(x) \right| < \varepsilon$$

с. разн. $\Rightarrow$ с. разн. с. к 0
однотонно с. разн. с. к 0

$$\sum_{k=n+1}^{\infty} |\varphi_k(x)| \leq \sum_{k=n+1}^{\infty} \max\{|\varphi_k(a)|, |\varphi_k(b)|\} = \text{с. разн. (т.е. монотон)}$$

$$\sum_{n=1}^{\infty} \varphi_n(x) = \text{с. разн. и с.к.}$$

872

D/P

778

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n + \sin x} ; 0 \leq x \leq 2\pi ; \quad 1) \frac{1}{n + \sin x} \rightarrow 0 ;$$

$$1) \alpha \frac{1}{n + \sin x} \leq \frac{1}{n-1}, \quad n \geq 2$$

$$2) \left| \frac{1}{n + \sin x} \right| = \rho_n(x), \quad \rho'_n = \frac{-1}{(n + \sin x)^2}$$

$$\left| \sum_{k=2}^n (-1)^k \right| \leq 1 \Rightarrow \text{с. разн. по пр. Дирхле.}$$

2789

$a_n \rightarrow \infty$ и т.д. $\sum_{n=1}^{\infty} \left| \frac{1}{a_n} \right|$ с.р. $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{x-a_n}$ с.р. а.с. и равном на комплексной плоскости, не содержащей a_n .

$$\left| \frac{1}{x-a_n} \right| = \frac{1}{|a_n|} \cdot \frac{1}{\left| 1 - \frac{x}{a_n} \right|} \leq \frac{1}{|a_n|} \cdot \frac{1}{1 - \frac{|x|}{|a_n|}} \leq \frac{1}{|a_n|} \cdot \frac{1}{1 - \frac{1}{2}} = \frac{2}{|a_n|}$$

напрямую $\Rightarrow 2 \cdot \sum \frac{1}{|a_n|}$.

2782

$$\sum_{n=1}^{\infty} \frac{(-1)^{[n]} \cdot \frac{1}{n}}{\sin(n+x)}; \quad 0 \leq x < \infty. \quad u_n = \frac{(-1)^{[n]} \cdot \frac{1}{n}}{\sqrt{1+\frac{x}{n}}}; \quad \left| \frac{1}{\sqrt{1+\frac{x}{n}}} \right| \leq 1$$

$\sum_{n=1}^{\infty} \frac{(-1)^{[n]}}{n}$ с.р. равн. $\Rightarrow$ чередующийся с.р. равном по Абелю.

2781

$$\sum_{n=1}^{\infty} a_n \text{ с.р.}; \quad \sum_{n=1}^{\infty} a_n e^{-nx} \text{ с.р. равн. в обл. } x \geq 0$$

$\sum_{k=1}^{\infty} a_n$ с.р. e^{-nx} монотонно с.р. $\Rightarrow$ ф.р. равн.

2780

$$\sum_{n=1}^{\infty} \frac{\cos \frac{2\pi n x}{3}}{2n^2+x^2}; \quad -\infty < x < \infty; \quad \left| \sum_{k=1}^{\infty} \cos \frac{2\pi k x}{3} \right| \leq \frac{1}{\left| \sin \frac{\pi}{3} \right|} = \frac{2}{\sqrt{3}}; \quad \frac{1}{2n^2+x^2} \leq \frac{1}{n^2} \text{ с.р.}$$

$\Rightarrow$ с.р. равн. по т. Дирхле

Н.Р.Р.Р.

Равном с.р. равен

$$1) f_n(x) = \frac{x}{1+n^2x^2}, \quad x \in [0; +\infty)$$

$$f'_n(x) = \frac{xn}{n+n^3x^2} = \frac{xn}{n(1+n^2x^2)} = \frac{2xn}{2n(1+n^2x^2)} < \frac{1}{2n} \xrightarrow{n \rightarrow \infty} 0 \text{ с.р. равном.}$$

$$2) f_n(x) = \frac{nx}{1+n^2x^2} \xrightarrow{x \in [0; 1]} f(x) \equiv 0 \quad f_n\left(\frac{1}{n}\right) = \frac{1}{1+1} = \frac{1}{2} \Rightarrow \text{нет равном. с.р.}$$

$$3) f_n(x) = x^n, \quad 0 \leq x \leq 1, \quad f_n(x) \rightarrow 0, \text{ если } x \in [0, 1) \quad f_n(x) \rightarrow 1, \text{ если } x=1$$

$$f(x) = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & x=1 \end{cases}$$

$$x^n < \varepsilon, \quad 0 \leq x < 1; \quad \ln x < \ln \frac{\varepsilon}{n}, \quad n > \frac{\ln \varepsilon}{\ln x}$$

$$\ln x < 0 \Rightarrow n > \frac{\ln \varepsilon}{\ln x} \Rightarrow N(\varepsilon, x) = \left\lceil \frac{\ln \varepsilon}{\ln x} \right\rceil + 1$$

$$\lim_{x \rightarrow 1-0} N(\varepsilon, x) = +\infty$$

не равн. с.р.

Пр. ~~напр.~~ $f_n \in C[0,1]$ и $f_n(x) \xrightarrow[n \rightarrow \infty]{} f(x)$, то $f(x) \in C[0,1]$.

4) $f_n(x) = \frac{1}{1+nx}$, $x \in [0,1]$; $f_n(x) = \begin{cases} 0, & x \neq 0 \\ 1, & x = 0 \end{cases}$

~~$f_n(\frac{1}{n}) = \frac{1}{1+1} = \frac{1}{2} \Rightarrow$~~ не равн. с.х.

5) $f_n(x) = 2nx \cdot e^{-n^2 x^2}$, $0 \leq x \leq 1$;

$f(x) \equiv 0$, $0 \leq x \leq 1$ (но не монотонно)

$f(\frac{1}{n}) = 2ne^{-1} \rightarrow +\infty$
 $n \rightarrow \infty$

6) $\sum_{n=1}^{\infty} x^{n-1} = 1 + x + \dots + x^n + \dots = \frac{1-x^{n+1}}{1-x} \xrightarrow{n \rightarrow \infty} \frac{1-x}{1-x} = 1$, $0 \leq x < 1 \Rightarrow \frac{1}{1-x}$.

Пр. ~~Однородн. зм. x^n~~ ; $f_n(x) \rightarrow f(x) \begin{cases} 0, & x \neq 1 \\ 1, & x = 1 \end{cases}$

$z_n(x) = \sum_{k=n+1}^{\infty} x^k = x^{n+1} \frac{1}{1-x}$

След. с.х. равн., то $f_n(x) \geq 0$

$z_n(x) \rightarrow \begin{cases} 0, & x \neq 1 \\ +\infty, & x = 1 \end{cases} \Rightarrow$ равн. не с.х.

Нес. ун. р. с.х. $\limsup_{n \rightarrow \infty} |z_n(x)| = 0$
и $\lim_{n \rightarrow \infty} z_n(x) = 0$

7) $\sum_{n=1}^{\infty} \frac{(-1)^n}{x^2+n}$, $x \in \mathbb{R}$ Две чл. гр. x будет знак по ~~знаку~~ $\begin{cases} \text{мон. } x > 0 \\ \text{уб. } x < 0 \\ (-1)^n \end{cases}$
 $\frac{1}{x^2+n+1} \leq \frac{1}{x^2+n}$; Т.е. несолет. с.х. есть;

$z_n(x) \leq \left| \frac{1}{x^2+n+1} \right| \leq \frac{1}{n+1} \xrightarrow{n \rightarrow \infty} 0$
равн. $\forall x$

Класс. с.х. групп. разн. ...

Пр. $u_n(x) \in R[a,b]$ и $\sum_{n=1}^{\infty} u_n(x) \xrightarrow[n \rightarrow \infty]{} S(x)$ $\Rightarrow S(x) \in R[a,b]$ и
 $\int_a^b \left(\sum_{n=1}^{\infty} u_n(x) \right) dx = \sum_{n=1}^{\infty} \int_a^b u_n(x) dx$ — норм. интегр. (только гр.)

Пр. $\sum_{k=1}^{\infty} u_k(x) < \infty$ holds в т. $x_0 \in [a,b]$

$u_k(x)$ уб. с.х. на $[a,b]$ и $\sum_{k=1}^{\infty} u_k'(x)$ с.х. равн. $\Rightarrow S(x)$ уб. с.х. на $[a,b]$ и

норм. интегр. $\frac{d}{dx} \left(\sum_{k=1}^{\infty} u_k(x) \right) = \sum_{k=1}^{\infty} u_k'(x) = S'(x)$ на с.х.

$$\sum_{n=1}^{\infty} \left(\frac{nx}{1+nx^2} - \frac{(n-1)x}{1+(n-1)x^2} \right), \quad 0 \leq x \leq 1 \quad \text{монотонно убывает}$$

$$S_k(x) = \sum_{n=1}^k \left(\frac{nx}{1+nx^2} - \frac{(n-1)x}{1+(n-1)x^2} \right) = \left(\frac{x}{1+x^2} + 0 + \frac{2x}{1+4x^2} - \frac{x}{1+x^2} + \dots - \frac{kx}{1+k^2x^2} - \frac{(k-1)x}{1+(k-1)x^2} \right)$$

$$= \frac{kx}{1+k^2x^2}; \quad S_k(x) \rightarrow 0 \quad 0 \leq x \leq 1, \quad \int_0^1 \frac{kx}{1+k^2x^2} dx = \frac{1}{2k} \int_0^1 \frac{d(1+k^2x^2)}{1+k^2x^2} =$$

$$= \frac{1}{2k} \ln(1+k^2x^2) \Big|_0^1 = \frac{1}{2k} \ln(1+k^2), \quad \lim_{k \rightarrow \infty} \frac{1}{2k} \ln(1+k^2) = 0;$$

$$S\left(\frac{1}{k}\right) = \frac{1}{1+1} = \frac{1}{2} \Rightarrow \text{равная на кр. нет; (монотонно не убывает)}$$

$$\sum_{n=1}^{\infty} \left(e^{-(n-1)x^2} - e^{-n^2x^2} \right), \quad S_n(x) = \sum_{k=1}^n (e^{-(k-1)x^2} - e^{-k^2x^2}) = 1 - e^{-x^2} + e^{-x^2} - e^{-4x^2} + \dots + e^{-n^2x^2} - e^{-(n+1)^2x^2}$$

$$= 1 - e^{-n^2x^2} \rightarrow 0, \quad x=0$$

$$S(x) \rightarrow 1, \quad x \neq 0$$

$$S_n(x) \rightarrow S(x) \in [0, 1], \quad \int_0^1 (1 - e^{-n^2x^2}) dx = x - \int_0^1 e^{-n^2x^2} dx;$$

$$\lim_{n \rightarrow \infty} \int_0^1 (1 - e^{-n^2x^2}) dx = 1 - \int_0^1 \lim_{n \rightarrow \infty} e^{-n^2x^2} dx = 1 - 0 = 1$$

52802

$$x \in [0, 1]; \quad \sum_{n=1}^{\infty} nx^n e^{-nx}, \quad 1) \lim_{n \rightarrow \infty} f_n(x) \text{ нет.}$$

$$2) \text{ равная на кр. нет; } f_n(x) \rightarrow f(x)$$

$$3) \lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \int_0^1 \lim_{n \rightarrow \infty} f_n(x) dx$$

$$1) n=y,$$

$$f_y(x) = yx^y e^{-yx}, \quad 0 \leq x \leq 1, \quad f_y(x) \rightarrow 0 \quad \forall x \text{ (неб. доказ.)}$$

$$\lim_{y \rightarrow \infty} \frac{yx^y}{e^{yx}} = \lim_{y \rightarrow \infty} \frac{x}{e^x} = 0$$

$$2) F_n(x) = |f_n(x)|, \quad F_n'(x) = n^x e^{-nx} - n^{x+1} x e^{-nx} = e^{-nx} n^x (1 - nx) = 0 \quad x = \frac{1}{n}; \quad \text{т.ма}$$

$$F_n\left(\frac{1}{n}\right) = n^{x-1} e^{-1} = \frac{1}{n^{1-x} e} \rightarrow 0 \quad 1-x > 0, \quad x > 1 \Rightarrow \text{равная на кр. нет}$$

$$3) n^x \int_0^1 x e^{-nx} dx = n^{x-1} \left(x e^{-nx} \Big|_0^1 - \int_0^1 e^{-nx} dx \right) = n^{x-1} \left(e^{-n} + \frac{1}{n} e^{-nx} \Big|_0^1 \right) = e^{-n} \left[\frac{1}{n^{1-x}} + 1 \right]$$

$$4) f_n(x) = n^2 x (1-x^2)^n, \quad 0 \leq x \leq 1 \quad \text{мон. на интервале}$$

$$f_n(x) \rightarrow f(x) = 0, \quad x \in [0, 1].$$

$$\int_0^1 n^2 x (1-x^2)^n dx = \frac{1}{2} n^2 \int_0^1 (1-x^2)^n d(1-x^2) = \frac{1}{2} n^2 \left[\frac{(1-x^2)^{n+1}}{-(n+1)} \right]_0^1 = \frac{1}{2(n+1)}$$

$$= -\frac{n^2}{2} \frac{(1-x^2)^{n+1}}{n+1} \Big|_0^1 = \frac{n^2}{2(n+1)} \rightarrow +\infty \Rightarrow \text{не устрем. нормен.}$$

$$\int_0^1 nx(1-x^2)^n dx = -\frac{n}{2(n+1)} \rightarrow -\frac{1}{2}, \quad \int_0^1 \sqrt[n]{n} x(1-x^2)^n dx = \frac{\sqrt[n]{n}}{2(n+1)} \rightarrow 0 \quad x < 1;$$

$$n^2 x(1-x^2)^n \quad Z_n(x) = n^2 x(1-x^2)^n$$

9P

22798

$$\theta(x) = \sum_{n=-\infty}^{+\infty} e^{-\pi n^2 x} \quad \text{оп. и сум. гар. при } x > 0; \quad e^{-\pi n^2 x} \leq e^{-\pi n^2} \rightarrow 0$$

$$\sum_{n=-\infty}^{+\infty} -\pi n^2 e^{-\pi n^2 x} = 2 \sum_{n=1}^{+\infty} -\pi n^2 e^{-\pi n^2 x} \quad \theta'(x) \rightarrow -\infty;$$

$$\sum_{n=-\infty}^{+\infty} n^2 e^{-\pi n^2 x}, \quad \text{реш. не устр. нормен. , это максимум } \sum_{n=-\infty}^{+\infty} n^2 e^{-\pi n^2 x}$$

2) не устр. нормен. гар. при $x > 0$.
(x_0 - пункт $\rightarrow x > 0$)
нормен.

22799

$$a) f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x}{n+x} \quad ; \quad x \in \mathbb{R} \quad \text{нормен. не устр. нормен.}$$

$$\left(\frac{(-1)^n x}{n+x} \right)' = \frac{(-1)^n n+x-x}{(n+x)^2} = \frac{(-1)^n n}{(n+x)^2} \quad ; \quad x \in \left[-\frac{1}{2}, \beta \right], \quad \beta > 0,$$

$$\text{при } x_0, \quad -(k_0+1) < x_0 < -k_0 \\ -(k_0+1) < a < k_0 < \beta < -k_0$$

$$\left| \frac{n}{(n+x)^2} \right| \leq \frac{n}{(n-1)^2} \rightarrow 0 \Rightarrow \text{нормен. при } \left[-\frac{1}{2}, +\infty \right)$$

$$\left| \frac{n}{(n+x)^2} \right| = \frac{n}{n^2 + 2nx + n^2} \leq \frac{1}{n^2 - 2n|x|} \leq \frac{n^2}{n^2 - 2n|a|} = \frac{1}{1 - 2|a|} \rightarrow 0 \Rightarrow \text{нормен. при } (-k_0+1, k_0)$$

нормен. $\left(\frac{x}{n+x} \right)'$ не устр. и $f'(x) = \sum_{n=1}^{\infty} \frac{n(-1)^n}{(n+x)^2}$ не устр. $\Rightarrow$ нормен. не устр.

$$b) \text{ Реш. } f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+x^2} \quad \text{нормен. не устр. при } x \rightarrow 0$$

$$f'(x) = \sum_{n=1}^{\infty} \frac{n^2 2nx - x|x|}{(n^2+x^2)^2}, \quad x \neq 0, \quad \varphi_n(x) = \frac{n^2 2nx - x|x|}{(n^2+x^2)^2} \leq \frac{n^2 + x^2}{n^4} \leq \frac{2n^2}{n^4} = \frac{2}{n^2}$$

(*) не устр. не устр. $\Rightarrow$ не устр. $\sum f'_n(x)$ не устр. $x \neq 0$

$$\text{т. } x=0: \quad \lim_{\Delta x \rightarrow 0} \frac{f(\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \left(\frac{1}{\Delta x} \sum_{n=1}^{\infty} \frac{1}{n^2 + (\Delta x)^2} \right) \quad \text{нормен. не устр.}$$

2) $\lim_{x \rightarrow 0} x = 0$ in $\mathbb{R}$.

$$\left| \frac{1}{2^{n+1}} \right| \leq \frac{1}{2^n} \Rightarrow \text{cr. pabai} \quad \text{u} \quad \lim_{x \rightarrow +\infty} \sum_{n=1}^{\infty} \frac{1}{2^{n+1}} x = 1.$$

гетероген. $\Rightarrow \lim_{x \rightarrow \infty} \sum_{n=1}^{\infty} \frac{x^2}{1+n^2 x^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

Kn. 25089

$$\sum_{k=1}^{\infty} c_k (x - x_0)^k = \text{срн. раз.}, \quad \sum_{k=1}^{\infty} c_k x^k \quad \text{IR} = \sup(X \mid \text{срн. раз. разл. срн. раз.})$$

Система рез, схем. в огней т. не будет излучать

Оператор эк. номер 5000: $E-R, R$, $\frac{1}{R} = \lim_{n \rightarrow \infty} \sqrt[n]{n!}$ - гр-а
 $E-R, R$
 $E-R, R$ - наим-Август

$$\frac{1}{R} = \lim_{n \rightarrow \infty} \left| \frac{C_{n+1}}{C_n} \right|$$

$$|x| < |x_0| \Rightarrow \sum c_k |x^k| = a \text{ s.c. c.k.}$$

Тег. $[E, R]$ - интервал с.к., $E-R \in E, R-E \in [E-R, R]$ равнозначен.

Лемма с.к. в области с.к. - л.в.-с.к. непрер. с.к. и.

$a_n \int x^n dx = \frac{a_n x^{n+1}}{n+1}$ / разные интервал - разные, с первой разницы - ин

Вспомог. рез. вкл. сж. можно получить зиг.

Результ. см. предпр. пред. совпадений с раз. из - изложения ст.
пред.

Разложение φ в степенные ряды
 $\sum_{k=0}^{\infty} c_k x^k = f(x)$ если имеет блочн. кан-во гранич. х. ^{необход.} _{достат.} x_0

$x=0$
~~не~~ применяя $\sum_{k=0}^{\infty} e_k x^k = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ - Бесскоечно раз
 0 - Бесско. разл.



$$\sum_{k=0}^{\infty} \frac{1}{k!} x^k$$

$x \neq 0$, не равно 0. $\lim_{x \rightarrow 0} 0$.

$\frac{1}{k!} x^k$ сходятся. радиус в формуле Тейлора (каждый член) $\rightarrow 0$
 $\lim_{k \rightarrow \infty} \frac{1}{k!} x^k = 0$

$$R = \lim_{n \rightarrow \infty} \frac{|C_n|}{|C_{n+1}|} - \text{радиус сходимости}$$

$$\sum_{k=0}^{\infty} C_k (x-x_0)^k, \quad |x-x_0| < R \text{ с.а.с.}; \quad \frac{x}{R} = y \Rightarrow |y| < 1$$

$$\sum_{k=0}^{\infty} \frac{x^k}{k^2} \quad [-1, 1], \quad \sum_{k=0}^{\infty} \frac{x^k}{k^2} \quad (-1, 1) - \text{пр-е Лейбница } x=1: \text{ рас-е.}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k} \quad [-1, 1], \quad \sum_{k=0}^{\infty} x^k \quad (-1, 1)$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n^p}, \quad p > 1 \text{ с.а.р.а.и. и } \lim_{n \rightarrow \infty} \frac{x^n}{n^p} \cdot \frac{(n+1)^p}{x^{n+1}} = \lim_{n \rightarrow \infty} \frac{1}{n^p} (n+1)^p = 1 \quad p > 0$$

$$x = -1; \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^p} \quad \left| \frac{(-1)^n}{n^p} \right| \leq \frac{1}{n^p} \quad p > 1 \text{ с.а.с.}$$

$$[-1, 1], \quad 0 < p \leq 1$$

$$[-1, 1], \quad p > 1$$

$$\sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{n} (x+1)^n$$

- 1) радиус с.а.
- 2) интервал
- 3) уз. концов

$$1) \frac{1}{R} = \lim_{n \rightarrow \infty} \sqrt[n]{C_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{3^n + (-2)^n}{n}} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{3^n + 2^n}{n}} = 3 \quad R = \frac{1}{3}$$

$$2) \left(-1 + \frac{1}{3}, -1 + \frac{1}{3} \right) = \left(\frac{2}{3}, \frac{4}{3} \right) \left(-\frac{4}{3}, -\frac{2}{3} \right) \quad 3) \sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{n} \left(\frac{2}{3} \right)^n = \sum_{n=1}^{\infty} \frac{(2)^n + \left(\frac{1+2}{3} \right)^n}{n} =$$

$$= \sum_{n=1}^{\infty} \frac{(-2)^n}{n} \left(\frac{1}{3} \right) + \sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{n} \left(-\frac{1}{3} \right)^n < \infty, \quad \sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{n} \left(\frac{1}{3} \right)^n < \infty$$

$$\sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{n}$$

linear $\alpha: (-4; 4);$

$$n! = \sum_{c=0}^n \binom{n}{c} \left(\frac{1}{2} \right)^n + O(n^2)$$

$$\in \sum_{i=1}^n (a_i^2 + b_i^2)$$

Chapters in order.

$(-1, 1)$; $\sum_{n=1}^{\infty} (1 + \frac{1}{2^n} + \frac{1}{n})$ не с. по крест. усл. $\Rightarrow$ для с. интерв. $(-1, 1)$.

22

2) Unterpunkt m-TH : $(\frac{1}{2}-2; \frac{1}{2}+1)$
 $(-\frac{3}{2}; \frac{5}{2})$ $(-1; 3)$

$$\frac{a_n}{a_{n+1}} = \left(1 + \frac{1}{2n+1}\right)^p = 1 + \frac{p}{2n+1} + \frac{p(p-1)}{2(2n+1)^2} + O\left(\frac{1}{n^2}\right), \quad 0 < b_n < \left(\frac{1}{\sqrt{2n+1}}\right)^p$$

$$a_n = \left(\frac{(2n-1)!!}{(2n)!!} \right)^p, \text{ лемма пр. Гаусса. Пусть } \begin{matrix} x=1 \\ p>0 \\ x=2 \end{matrix} \text{ и } (p>2 \text{ см. ад.})$$

IP $\exists x (x \in \omega) \wedge P \leq 2^{\aleph_0}$

2827 16 мин

$$f(x+h) - f(x) = hf'(x) + \frac{h^2}{2!} f''(x) + \dots = \sum_{n=1}^{\infty} \frac{f^{(n)}(x)}{(n+1)!} \left(1 + \frac{1}{2} + \dots + \frac{1}{n}\right) x^n$$

2829

$$\sum_{n=2}^{\infty} \frac{(1+2\cos \frac{\pi n}{4})^n}{n n!} x^n; \quad \lim_{n \rightarrow \infty} \frac{(1+2\cos \frac{\pi n}{4})}{\sqrt[n]{n n!}} = \frac{3}{1} = \frac{1}{R} \Rightarrow R = \frac{1}{3};$$

Интервал: $(-\frac{1}{3}; \frac{1}{3})$;

$$\sum_{n=2}^{\infty} \frac{(1+2\cos \frac{\pi n}{4})^n}{n n!} \frac{1}{(\pm 3)^n}; \quad \sum_{k=1}^{\infty} \frac{(1+2\cos 2\pi k)^{8k}}{8k} \frac{1}{3^{8k}} = \sum_{k=1}^{\infty} \frac{1}{8k+8}$$

$$\frac{1}{8k+8} > \frac{1}{k+8} > 0$$

$$\sum_{n=2}^{\infty} \left| \frac{(1+2\cos \frac{\pi n}{4})^n}{n!} \right| \cdot \frac{1}{3^n} \leq \sum_{n=2}^{\infty} \left(\frac{1+\sqrt{2}}{3} \right)^n < \sum_{n=1}^{\infty} \left(\frac{1+\sqrt{2}}{3} \right)^n < \frac{1+\sqrt{2}}{3} \cdot \frac{1}{1 - \frac{1+\sqrt{2}}{3}} = \frac{1+\sqrt{2}}{2-\sqrt{2}} < 5.$$

2) эк., где берем

2839 (сб. 19.?)

2853

$$\sinh^2 x = \left(\frac{x}{1} - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right) \left(\frac{x}{1} - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right) \left(\frac{x}{1} - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right)$$

$$\frac{1}{4} \sinh x - \frac{1}{4} \sinh^2 x = \frac{1}{4} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} - \frac{1}{4} \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \frac{1}{4} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

2859

$$\frac{12-5x}{6-5x-x^2} = \frac{12-5x}{-(x^2+5x-6)} = \frac{12-5x}{-(x+6)(x-1)} = - \left(\frac{-6}{x+6} + \frac{1}{x-1} \right) =$$

$$= 6(x+6)^{-1} + (x-1)^{-1} = 6 \left(1 + (x+5)(-1) + \frac{2}{2!} (x+5)^2 + \dots \right)$$

$$= (1+(x+5))^{-1} - (1+(x-1))^{-1} = - \left(1 - (x-1) + \frac{1}{2!} (x-1)^2 + \dots \right)$$

$$\textcircled{e} 5 - \frac{(x+5)^2}{(x+5)(x-1)} + \frac{2}{2!} (- (x+5)^2 + (x-1)^2) + \dots$$

2898

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right| \text{ и } l = \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right|, \quad 0 < l < R < \infty, \quad \sum_{n=0}^{\infty} a_n x^n, \quad R < l < \infty$$

$$\text{т.е. } R = \frac{1}{\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}}, \text{ где.}$$

$$R = \lim_{n \rightarrow \infty} \frac{a_n}{a_{n+1}}$$

$$\forall n > m \quad l \cdot (1-\varepsilon) < \left| \frac{a_n}{a_{n+1}} \right| < l \cdot (1+\varepsilon), \quad \frac{1}{1+\varepsilon_1} = 1 - \frac{\varepsilon}{2}, \quad \frac{1}{1-\varepsilon_2} = 1 + \frac{\varepsilon}{2}$$

$$\frac{1}{l} \cdot \frac{1}{1+\varepsilon_1} < \left| \frac{a_{n+1}}{a_n} \right| < \frac{1}{l} \cdot \frac{1}{1-\varepsilon_2}$$

$$l_1 \cdot (1 - \frac{\varepsilon}{2}) < \frac{|a_{n+1}|}{|a_n|} < l_1 \cdot (1 + \frac{\varepsilon}{2}), \quad \frac{|a_n|}{|a_m|} = \frac{|a_n|}{|a_{n-1}|} \cdot \frac{|a_{n-1}|}{|a_{n-2}|} \cdot \dots \cdot \frac{|a_{m+1}|}{|a_m|} <$$

$$< (l_1 \cdot (1 + \frac{\varepsilon}{2}))^{n-m}$$

$$\frac{|a_n|^{\frac{1}{n}}}{L} < \left(\frac{|a_n|}{L^n}\right)^{\frac{1}{n}} (1 + \frac{\varepsilon}{2}), \quad \frac{|a_n|^{\frac{1}{n}}}{L} > \left(\frac{|a_n|}{L^n}\right)^{\frac{1}{n}} \cdot (1 - \frac{\varepsilon}{2});$$

$$\left(\frac{|a_n|}{L^n}\right)^{\frac{1}{n}} < 1 + \frac{\varepsilon}{1 - \frac{\varepsilon}{2}}, \quad \left(\frac{|a_n|}{L^n}\right)^{\frac{1}{n}} > 1 - \frac{\frac{\varepsilon}{2}}{1 - \frac{\varepsilon}{2}} \quad n \geq n_0;$$

$$\frac{|a_n|^{\frac{1}{n}}}{L} < 1 + \varepsilon, \quad \frac{|a_n|^{\frac{1}{n}}}{L} > 1 - \varepsilon, \quad L_1 \cdot (1 - \varepsilon) \leq \lim_{n \rightarrow \infty} |a_n|^{\frac{1}{n}} \leq L_2 \cdot (1 + \varepsilon)$$

$$\frac{1}{L(1+\varepsilon)} \leq R \leq \frac{1}{L(1-\varepsilon)}, \quad \frac{L}{1+\varepsilon} \leq R \leq \frac{L}{1-\varepsilon}$$

$$L \leq R \leq L$$

28

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots = \sum_{k=0}^{\infty} \frac{x^k}{k!} \quad x \in (-\infty, +\infty) \text{ разлож. в } (-\infty, +\infty)$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} \quad \text{для } |x| < 1$$

$$(1+x)^{\alpha} = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \dots + \frac{\alpha(\alpha-1)\dots(\alpha-k+1)}{k!} x^k + \dots$$

$$-1 < x < 1$$

Крит. и дост. усл. (признак б. разл.)

$$1) f(x) \in C_{\infty}$$

$$2) R_n \rightarrow 0, R_n - \text{ост. член в разложении Тейлора}$$

2819

$$\text{Разл. по } f(x) = \frac{1}{a-x}, \quad a \neq 0 \quad 1) \frac{1}{a-x} = \frac{1}{a} \cdot \frac{1}{1 - \frac{x}{a}} \quad \left|\frac{x}{a}\right| < 1 \Rightarrow \sum_{k=0}^{\infty} \left(\frac{x}{a}\right)^k = \sum_{k=0}^{\infty} \frac{x^k}{a^{k+1}}$$

$$1) \text{ не разл. по } x$$

$$2) \text{ не разл. по } (x-b)$$

$$3) \text{ не разл. по } \frac{1}{x}$$

$$2) \frac{1}{a-x} = \frac{1}{a-b-(x-b)} = \frac{1}{a-b} \cdot \frac{1}{1 - \frac{(x-b)}{a-b}} = \frac{1}{a-b} \sum_{k=0}^{\infty} \left(\frac{x-b}{a-b}\right)^k$$

$$\sum_{k=0}^{\infty} \frac{(x-b)^k}{(a-b)^{k+1}}$$

$$q = \left|\frac{x-b}{a-b}\right| < 1$$

$$-1 < \frac{x-b}{a-b} < 1,$$

$$2b-a < x < a = (2b-a, a) - \text{интервал сдв.}$$

$$3) \frac{1}{a-x} = \frac{1}{x} \cdot \frac{1}{\frac{a}{x} - 1} = \frac{1}{x} \cdot \frac{1}{1 - \frac{a}{x}} \Rightarrow \frac{1}{x} \sum_{k=0}^{\infty} \left(\frac{a}{x}\right)^k = \sum_{k=0}^{\infty} \frac{a^k}{x^{k+1}}$$

$$q = \left|\frac{a}{x}\right| < 1$$

$|x| > |a|$ - условие: бесконечное сдвиг.

2840

$f(x) = \ln(x)$ на от. $(x-1)$

$$\ln(1+(x-1)) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-1)^n}{n!}, \text{ радиус. ст. : } \frac{1}{|x-1|} < 1, \quad 0 < x < 2$$

лг. при $x=2$: $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k} = \ln 2$

$x=0$: разв. $\Rightarrow$ не выводится. (лимит. $x=1$)

$$\sinh x = \frac{e^x - e^{-x}}{2}, \text{ (умнож.)}$$

$$\sinh x = \frac{1}{2} \sum_{k=0}^{\infty} \left(\frac{x^k}{k!} - \frac{(-x)^k}{k!} \right) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{x^k (1 - (-1)^k)}{k!} = \sum_{k=1}^{\infty} \frac{x^{2k-1}}{(2k-1)!}$$

только нечетные k : x^{2k-1}
или: x - нечетная степень

$$\cosh x = \frac{e^x + e^{-x}}{2} = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} \quad R = \lim_{n \rightarrow \infty} \frac{a_n}{a_{n+1}} = \lim_{n \rightarrow \infty} \frac{(2n+2)!}{(2n+1)!} = +\infty$$

$-\infty < x < +\infty$

$\forall x \in \mathbb{R}$ разв. ст. аддон., а на любом конечном отрезке ст. разв. не будет мен. не конечн. ст. (разв. неогранич.)

$$\sinh x \in C^\infty(\mathbb{R})$$

2853

$$\sinh^3 x = \frac{3}{4} \sinh x - \frac{\sinh 3x}{4}, \quad \sinh(x+2x) = \sinh x \cosh 2x + \cosh x \sinh 2x = \sinh x (1 + 2 \sinh^2 x) + 2 \cosh x \sinh x$$

$$\odot \sinh x - 2 \sinh^3 x + 2 \sinh x (1 + 2 \sinh^2 x) = 3 \sinh x - 4 \sinh^3 x$$

$$\sinh^5 x = \frac{1}{4} \sum_{k=0}^{\infty} (-1)^k x^{2k+1} \left(\frac{1}{(2k+1)!} - \frac{3}{(2k+1)!} \right) = \frac{1}{4} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} (1 - 3) \quad R = +\infty$$

$\forall x \in \mathbb{R}$ разв. ст. аддон., на любом сегм. ст. разв. $x \in (-\infty, +\infty)$

$$\sinh^5 x \in C^\infty(\mathbb{R})$$

2855

$$f(x) = \frac{1}{(1-x)^2}, \quad \text{аддон. ст. } x \neq 1$$

$$\frac{d}{dx} \left(\frac{1}{1-x} \right) = \frac{1}{(1-x)^2} = \sum_{k=0}^{\infty} \frac{1}{1-x} x^k = \sum_{k=0}^{\infty} \frac{1}{1-x} x^k = \sum_{k=0}^{\infty} x^k = \sum_{k=0}^{\infty} x^k$$

(разв. ст. аддон. ст. $x \neq 1$)
или: $k-1 = k' \Rightarrow k = 1+k'$

$$R = \lim_{n \rightarrow \infty} \frac{a_n}{a_{n+1}} = \lim_{n \rightarrow \infty} \frac{n+1}{n+2} = 1.$$

$\Rightarrow -1 < x < 1$ - вып. ст.

Ур. зм.: $x=1$ - неБ,
 $x=-1$, раскоз. (неБ. ур. СК-)

$$\frac{d}{dx} \left(\frac{1}{(1-x)^2} \right) = \frac{2}{(1-x)^3} = \frac{d}{dx} \left(\sum_{k=0}^{\infty} (k+1)x^k \right) = \sum_{k=1}^{\infty} k(k+1)x^{k-1} = \sum_{k=0}^{\infty} (k+2)(k+1)x^k$$

275

$$\text{QUESTION } \sum_{k=0}^{\infty} (k+3)(k+2)(k+1)x^k = ?$$

$$\sum_{k=0}^{\infty} (k+1)(k+2)(k+3)(k+4) x^k = ?$$

52857

$$f(x) = \ln \sqrt{\frac{1+x}{1-x}}, \quad D(f(x)) : \begin{cases} x \neq 1 \\ x \neq -1 \end{cases} \quad \frac{1+x}{1-x} > 0$$

$$11 \quad \frac{1}{2} (\ln(1+x) - \ln(1-x)) \quad (-1 < x < 1)$$

$$= \frac{1}{2} \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots - (-1)^{n-1} \frac{x^n}{n} + \dots - \left(x + \frac{x^2}{2} + \frac{x^3}{3} - \dots \right) \right)$$

$$\textcircled{iii} \sum_{k=0}^{\infty} \frac{x^{2k+1}}{2k+1} \cdot R=1; \quad -1 < x < 1, \text{ lim. ed.} \quad + \rightarrow \textcircled{iv}$$

1. Ели е. ред е. 7) кога изврш. одним образом.

52858

$$f(x) = \frac{x}{1+x-2x^2}, \quad x_{1,2} = 1, -\frac{1}{2}$$

$$f(x) = \frac{x^5}{2(1-x)} + \frac{-2(x-1)(x+\frac{1}{2})}{x+\frac{1}{2}} = \frac{Ax + \frac{1}{2}A + 2B - 2Bx}{2(1-x)(x+\frac{1}{2})} = \frac{1}{3(1-x)} - \frac{1}{3(2x+1)} = \frac{1}{3} \sum_{k=0}^{\infty} (1-x)^k - \frac{1}{3} \sum_{k=0}^{\infty} (-2x)^k$$

$|x| < 1$
 $|2x| < 1$
 $|x| < \frac{1}{2}$

$$\textcircled{E} \quad \frac{1}{3} \sum_{k=0}^{\infty} x^k (1-2^k (-1)^k) \quad \lim_{n \rightarrow \infty} \frac{1-2^{k+1} (-1)^{k+1}}{1-2^{k+1} (-1)^{k+1}} = \frac{1}{2} ; \quad \frac{1}{3} \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k (1-2^k (-1)^k) = \frac{1}{3}$$

реш. $x = \frac{1}{2}$.

52759

$$f(x) = \arctan x, \quad f'(x) = \frac{1}{1+x^2} = \sum_{k=0}^{\infty} (-1)^k x^{2k}$$

$$|x^2| < 1 \Rightarrow |x| < 1$$

$$f(x) = \int_0^x f'(t) dt = \int_0^x \sum_{k=0}^{\infty} f^{(k)}(1) t^k dt =$$

На любой сеп.
в к/с. разн.

$$= \sum_{k=0}^{\infty} (-1)^k \int_0^x t^{2k} dt = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} = \arctan x$$

Реш. с разн.
указ. п. должно
нормально быть.

1876

$f(x) = \frac{1}{1-x}$ разн. по у. $(\frac{1}{x})$ $f(x) = -\sum_{k=0}^{\infty} \frac{1}{x^{k+1}}$, ага, ок. $|\frac{1}{x}| < 1$

$|x| > 1$ бн.
ег. зам.
аналитич.

2877

$f(x) = \ln x$ на сеп. $\left| \frac{(x-1)^k}{x^{k+1}} \right| < 1$, $-1 < \frac{x-1}{x+1} < 1$, $x > 0$

$\ln x = \ln \frac{1 + \frac{x-1}{x+1}}{1 - \frac{x-1}{x+1}} = \ln \left(\frac{1 + \frac{x-1}{x+1}}{1 - \frac{x-1}{x+1}} \right) = \ln \left(1 + \frac{x-1}{x+1} \right) - \ln \left(1 - \frac{x-1}{x+1} \right) =$

$\sum_{k=0}^{\infty} \int (k+1)(x+1)^{k+2} x^{k+1} \left(\sum_{k=0}^{\infty} \frac{d}{dx} \left((k+2)(x+1)^{k+1} \right), \frac{d}{dx} \left(\sum_{k=0}^{\infty} (k+2) \frac{(x+1)^{k+1}}{x^{k+1}} \right) \right)$

2902

$\int_0^x \frac{dt}{\sqrt{1-t^2}} = \int_0^x \frac{dt}{(1-t^2)^{1/2}} = \int_0^x (1-t^2)^{-1/2} dt =$
 $= \int_0^x \left(1 + \frac{1}{2}t^2 + \frac{(1)(-3)}{2}t^4 + \dots \right) dt =$
 $= \left(t + \frac{1}{6}t^3 + \frac{3}{2} \cdot \frac{t^5}{5} + \dots \right) \Big|_0^x = x + \frac{1}{6}t^3 + \frac{1}{28}t^5 + \dots$

$\frac{d}{dx} \left(\sum_{k=0}^{\infty} (k+3) x^{k+2} \right)$
 $\frac{d}{dx} \sum_{k=0}^{\infty} x^{k+1}$
 $\frac{1}{1-x} = 1 - x - x^2$
 $\frac{1}{(1-x)^2} = 1 - 2x$
 $+ \frac{2}{(1-x)^3} = 2$
 $\frac{6}{(1-x)^4}$

2904

$\int_0^x \frac{\arctan x}{x} dx = \int_0^x \frac{\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}}{x} dx =$
 $= \int_0^x \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{2k+1} dx = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)^2}$

2907

$F(x) = \frac{x^3}{3} + \frac{x^5}{5} - \dots = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} \left(\sum_{k=1}^{\infty} \right)$
 $F'(x) = 1 - \frac{x^2}{2} + x^4 - \dots = \frac{1}{1-x^2}$, $F(0) = 0$; $F(x) = \int_0^x \frac{dt}{1-t^2} = \frac{\arctan x}{|x| \leq 1}$

2912

$F(x) = x - 4x^2 + 9x^3 - 16x^4 + \dots$, $\int_0^x F(x) = \sum_{k=2}^{\infty} (-1)^k \frac{(k-1)x^k}{k}$

$$\int_0^x F(t) dt = \frac{1}{2}x^2 - (1+\frac{1}{3})x^3 + (2+\frac{1}{4})x^4 - (3+\frac{1}{5})x^5 + \dots = x + (-x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \dots) - x^3(1-2x+3x^2-\dots) = x - \ln(1+x) - x^3(x-x^2+x^3-\dots)' = x - \ln(1+x) - x^3(\frac{x}{1+x})' = x - \ln(1+x) - \frac{x^3}{(1+x)^2} \quad (|x| < 1), \quad F(x) = \sum_{n=1}^{\infty} (-1)^{n-1} n^2 x^n = (x - \ln(1+x) + \frac{x^3}{(1+x)^2})' = \frac{x(1-x)}{(1+x)^3} \quad (|x| < 1).$$

2809

$$(\frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \dots)' = (\sum_{k=1}^{\infty} \frac{x^k}{k(k+1)})' = \sum_{k=1}^{\infty} \frac{x^{k-1}}{k+1};$$

$$F'(x) = \frac{1}{1 \cdot 2} + \frac{x}{2 \cdot 3} + \frac{x^2}{3 \cdot 4} + \dots = \sum_{k=1}^{\infty} \frac{x^{k-1}}{k+1}$$

$$\ominus -\frac{1}{x} + \frac{1}{x^2} + (x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots) = -\frac{1}{x} + \frac{1}{x^2} (-\ln(1-x)) \quad (0 < |x| < 1)$$

$$F(0) = 0, \quad F(x) = \int_0^x F'(t) dt = 1 + \frac{1-x}{x} \ln(1-x), \quad 0 < |x| < 1.$$

$$x=0 \Rightarrow 0; \quad x=1: \text{ некорректно}; \quad x=-1: 1-2\ln 2, \quad x=1 \Rightarrow 1$$

$$-\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} - \frac{1}{3 \cdot 4} + \dots = -(1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) - (\frac{1}{3} - \frac{1}{4}) + \dots$$

$$\ominus 2 \cdot (-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots) + 1 = 1 - 2\ln 2.$$

2839

$$1) f(x) = \sum_{n=0}^{\infty} a_n x^n, \quad \ln |a_n| < M \quad n=1, 2, \dots, \quad M = \text{const}$$

1) $f(x)$ равномерно вып. в $\forall \tau, a$

$$2) f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \quad (|x| < +\infty)$$

$$2) \text{ Пусть } f(x) \in C^{(\infty)}(a, b) \text{ и } |f^{(n)}(x)| \leq C^n \quad (n=0, 1, \dots) \text{ при } x \in (a, b).$$

З-то, что $f(x)$ равна к со. разг $\sum_{n=0}^{\infty} \frac{a_n}{n!} (x-a)^n \quad (x \in (a, b))$, и к. в мн-ве (a, b) .

$$3) \text{ Пусть } f(x) \in C^{(\infty)}[-1, 1] \text{ и } f^{(n)}(x) \geq 0 \quad (n=0, 1, \dots) \quad x \in [-1, 1]$$

$$\text{З-то, что в мн-ве } (-1, 1) \quad f(x) = \sum_{n=0}^{\infty} a_n x^n.$$

$$|a_n| < \frac{M}{n!}, \quad |a_n (x-x_0)^n| < \frac{M}{n!} (2N)^n; \quad \text{т.е. } \sum_{n=0}^{\infty} a_n (x-x_0)^n \text{ к. а. в. к. в. } x \in (-\infty, +\infty).$$

$$f^{(m)}(x) = \sum_{n=m}^{\infty} n(n-1)\dots(n-m+1) a_n (x-x_0)^{n-m} =$$

$$= \sum_{n=m}^{\infty} \frac{n!}{(n-m)!} a_n (x-x_0)^{n-m}, \quad m=1, 2, \dots$$

2956

$$\sum_{n=1}^{\infty} \frac{2^n (n!) x^n}{n!}; \quad \sum_{n=0}^{\infty} \frac{e^{n+1} x^{n+1}}{n!} \Rightarrow \frac{C_n}{C_{n+1}} \Rightarrow \frac{n+1}{2} \rightarrow +\infty$$

$$\left(\frac{2^n x^n}{n!} \right)' = \frac{2^n (n+1) x^n}{n!}; \quad \sum_{n=0}^{\infty} \frac{2^{n+1} x^{n+1}}{n!} = 2x \sum_{n=0}^{\infty} \frac{2^n x^n}{n!} = 2x e^{2x} \quad R = \infty$$

$$(2x e^{2x})' = 2(e^{2x}) (2 + 2x \cdot 2) = e^{2x} (2 + 4x)$$

$$\sum_{n=0}^{\infty} \frac{2^{n+1}}{n!} = \frac{2e^{2x} 4e^2}{2} = e^2 + 2e^2 = 3e^2$$

Умножение логарифмическим методом

2984

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right) = \frac{1}{2} \left(\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \dots - \frac{1}{2k+1} \right) = \frac{1}{2} \left(1 - \frac{1}{2k+1} \right) \rightarrow \frac{1}{2}$$

→ 1/2

2965

$$f(x) = \frac{x \sinh x}{1 - 2x \cosh x + x^2} \quad \text{---} \quad \cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\begin{aligned} \Rightarrow \frac{x \frac{e^x - e^{-x}}{2}}{1 - 2x \frac{e^x + e^{-x}}{2} + x^2} &= \frac{1}{2} \frac{x(e^x - e^{-x})}{(x - e^x)(x - e^{-x})} = \frac{1}{2} \frac{e^x}{x - e^x} - \frac{1}{2} \frac{e^{-x}}{x - e^{-x}} = \\ &= \frac{1}{2} \left(-\frac{1}{\frac{x}{e} - 1} - \frac{1}{\frac{x}{e^{-1}} - 1} \right) = \frac{1}{2} \left(\sum_{n=0}^{\infty} (x^n e^{x(n+1)} - x^n e^{-x(n+1)}) \right) = \frac{1}{2} \sum_{n=0}^{\infty} x^n \sinh(2n) \end{aligned}$$

$x e^x < 1 \quad x < e^{-x}$

Пример. 6. Найти сумму ряда чисел $\sin kx$ при $x \in \mathbb{R}$

$$\sum_{k=1}^n \sin kx = \frac{2 \sin \frac{nx}{2} \sin \frac{(n+1)x}{2}}{2 \sin \frac{x}{2}} = \frac{\cos \frac{x}{2} - \cos \frac{(n+1)x}{2}}{2 \sin \frac{x}{2}} = \frac{2 \sin \frac{(n+1)x}{2} \sin \frac{-nx}{2}}{2 \sin \frac{x}{2}}$$

$$\cos \alpha - \cos \beta = 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$S_n = \sum_{k=0}^n \frac{S_k}{n+1} = \sum_{k=0}^n \frac{\sin \frac{(k+1)x}{2} \sin \frac{kx}{2}}{(n+1) \sin \frac{x}{2}} = \frac{1}{(n+1) \sin \frac{x}{2}} \sum_{k=0}^n (\cos \frac{x}{2} - \cos \frac{(k+1)x}{2}) = S_n$$

$$S_n = \frac{1}{2} + \sum_{k=1}^n \cos kx = \frac{1}{2} + \frac{1}{2 \sin \frac{x}{2}} (\sin x(n+1/2) - \sin \frac{x}{2}) = \frac{\sin x(n+1/2)}{2 \sin \frac{x}{2}} \quad \text{---} \quad \text{логарифмический метод}$$

$$S_n = \sum_{k=0}^n \frac{1}{n+1} (S_k) = \frac{1}{n+1} \sum_{k=0}^n \frac{\sin x(k+1/2)}{2 \sin \frac{x}{2}} = \frac{1}{(n+1) \sin \frac{x}{2}} \sum_{k=0}^n \sin x(k+1/2) \sin \frac{x}{2} \quad \text{---} \quad \text{---}$$

$$2 \cos \alpha \sin \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta); \quad \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\Rightarrow \frac{1}{(n+1) \sin \frac{x}{2}} \sum_{k=0}^n \cos kx = \frac{1}{(n+1) \sin \frac{x}{2}} (\cos x(n+1) - \cos 0) = \frac{\cos 0 - \cos x(n+1)}{(n+1) \sin \frac{x}{2}} = \frac{1 - \cos x(n+1)}{(n+1) \sin \frac{x}{2}}$$

$$= \frac{\sin^2 \frac{\pi n}{2}}{2(n+1) \sin^2 \frac{\pi}{2}} \int_{-\pi}^{\pi} f(x+1) \frac{\sin^2 \frac{\pi(x+1)}{2}}{2(n+1) \sin^2 \frac{\pi}{2}} dx \xrightarrow{L.H.V} f(x) \quad \text{Per.}$$

Алгоритм для $\sum u_n \rightarrow A(C)$, но $u_n = O(n)$.

$$S_n = \frac{1}{n+1} \sum_{k=0}^n s_k \rightarrow A, \quad \frac{(n+1)}{n} S_n \rightarrow A \quad \frac{(n+1) \cdot \frac{1}{n+1} S_n - n S_{n-1}}{n} = \frac{S_n}{n} \rightarrow 0$$

$$\Rightarrow \frac{u_n}{n} = \frac{S_n}{n} - \frac{(n-1)}{n} S_{n-1} \rightarrow 0 \Rightarrow (u_n = O(n)).$$

Д) покажем, что $\sum u_n \rightarrow A(PA)$, но не $\rightarrow A$. ($u_n = O(n)$)
 и u_n не PA



$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = \sum_{n=1}^{\infty} \left(\frac{1}{2n} - \frac{1}{n+1} + \frac{1}{2(n+2)} \right) = \sum_{n=1}^{\infty} \left(-\frac{1}{2n} + \frac{1}{n+1} - \frac{1}{2(n+2)} \right) =$$

$$= \left(-\frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{2} + \frac{1}{4} - \frac{1}{2} + \frac{1}{5} + \frac{1}{2} - \frac{1}{4} \right) + \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \frac{1}{10} - \frac{1}{11} + \dots \right) = -\frac{1}{4}$$

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)(n+3)} = \left(\frac{1}{2} - \frac{3}{4} \right) = \sum_{n=1}^{\infty} \left(-\frac{1}{2(n+1)} + \frac{2}{n+2} - \frac{3}{2(n+3)} \right) =$$

$$\frac{n}{(n+1)(n+2)(n+3)} = \frac{A_1}{n+1} + \frac{A_2}{n+2} + \frac{A_3}{n+3} \Rightarrow -1 = 2A_1, \quad A_1 = -\frac{1}{2}, \quad A_2 = 2, \quad A_3 = -\frac{3}{2}$$

$$\sum_{n=1}^{\infty} \left(-\frac{1}{2(n+1)} + \frac{2}{n+2} - \frac{3}{2(n+3)} \right) = \left(-\frac{1}{2} + \frac{2}{3} - \frac{3}{2} + \frac{1}{4} - \frac{1}{2} + \frac{2}{5} - \frac{3}{2} + \frac{1}{6} + \frac{1}{2} - \frac{1}{4} + \frac{2}{6} - \frac{3}{2} + \frac{1}{8} + \frac{1}{4} - \frac{1}{6} + \frac{2}{8} - \frac{3}{2} + \frac{1}{10} + \frac{1}{5} - \frac{1}{10} + \dots \right) = -\frac{1}{4}$$

$$\frac{n}{(n+1)(n+2)(n+3)} = \frac{1}{n+1} \left(\frac{n}{(n+2)(n+3)} \right) = -\frac{1}{4} + \frac{2}{4} + \frac{2}{n+2} - \frac{3}{2} \frac{1}{n+3} \rightarrow -\frac{1}{4}$$

$\exists f \in C(\mathbb{C}) (|z| < R) \Rightarrow$ на K с. рабн. $\Rightarrow$ по теор. Г. Г. с. рабн. $\Rightarrow$ f непрерывна
и можно и по определению
групп. по 2.

$$W = w(z) = u(x, y) + i v(x, y);$$

$$e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!}, \quad e^z = e^{x+iy} = e^x e^{iy} = e^x (\cos y + i \sin y), \quad e^{iy} = 1 + iy - \frac{y^2}{2!} - i \frac{y^3}{3!} + \dots$$

$$e^{iy} = \cos y + i \sin y \quad \text{— группа по определению}$$

$$e^{-iy} = \cos y - i \sin y \Rightarrow \cos y = \frac{e^{iy} + e^{-iy}}{2}, \quad \sin y = \frac{e^{iy} - e^{-iy}}{2i}$$

Сгенерируем ряды в комплексной области

$$\sum_{k=1}^{\infty} \cos kx = \sum_{k=1}^{\infty} \frac{e^{ikx} + e^{-ikx}}{2} = \frac{1}{2} \left(\frac{e^{ix} - e^{i(n+1)x}}{1 - e^{ix}} \right) + \frac{1}{2} \left(\frac{e^{-ix} - e^{-i(n+1)x}}{1 - e^{-ix}} \right) \quad (*)$$

$$\cos kx = \frac{e^{ikx} + e^{-ikx}}{2}$$

генерируем $e^{ix/2}$ и $e^{-ix/2}$

$$\begin{aligned} & \frac{1}{2i} \frac{e^{ix} - e^{i(n+1)x}}{e^{ix/2} - e^{i(n+1)x/2}} + \frac{1}{2i} \frac{e^{-ix} - e^{-i(n+1)x}}{e^{-ix/2} - e^{-i(n+1)x/2}} = \frac{\sin^{n+1/2} x}{2 \sin^{n+1/2} x} - \frac{1}{2} \Rightarrow \\ & \Rightarrow S_n(x) = \dots \end{aligned}$$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \quad (*)$$

$$|z| < 1, \quad r = \sqrt{x^2 + y^2} = |z|$$

$$z = (x+iy) = r(\cos \varphi + i \sin \varphi); \quad z^n = r^n(\cos n\varphi + i \sin n\varphi) \quad \text{— по формуле Муавра}$$

$$\sqrt[n]{z} = \sqrt[n]{r} \left(\cos \frac{\varphi + 2\pi k}{n} + i \sin \frac{\varphi + 2\pi k}{n} \right), \quad k=0, 1, \dots, n-1.$$

$$\sum_{n=0}^{\infty} z^n (\cos \varphi + i \sin \varphi) = \frac{1 - r^n \cos \varphi + i r^n \sin \varphi}{1 - r(\cos \varphi + i \sin \varphi)} = \frac{1 - r(\cos \varphi - i \sin \varphi)}{(1 - r \cos \varphi)^2 + r^2 \sin^2 \varphi} \quad (**)$$

$$\frac{1 - r^2 \cos \varphi}{1 - 2r \cos \varphi + r^2} = \sum_{n=0}^{\infty} r^n \cos n\varphi; \quad \operatorname{Im} \frac{z i \sin \varphi}{1 - 2r \cos \varphi + r^2} = \sum_{n=0}^{\infty} r^n \sin n\varphi$$

$$u = f(x, y) \\ v = \varphi(x, y)$$

$$f(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$$

$$f(y) = \sum_{k=0}^{\infty} \frac{y^k}{k!} = e^y$$

$$f(x+iy) = \sum_{n=0}^{\infty} \sum_{j=0}^n \frac{(x+iy)^n}{j!(n-j)!} = \sum_{n=0}^{\infty} \frac{(x+iy)^n}{n!} = e^{x+iy}$$

или по определению
групп. по 2.

$$\sum_{n=0}^{\infty} \frac{\cos nx}{n!}$$

$$z = \cos y + i \sin y$$

$$z^n = (\cos ny + i \sin ny)$$

$$\operatorname{Re} \sum_{n=0}^{\infty} \frac{(\cos nx + i \sin nx)}{n!} = \operatorname{Re} \sum_{n=0}^{\infty} \frac{z^n}{n!} = \operatorname{Re} \frac{1}{1-z} z = e^{\cos x} e^{i \sin x} = e^{\cos x} (\cos \sin x + i \sin \sin x)$$

$$e^{\cos x} \cos(\sin x)$$

Ex 3017

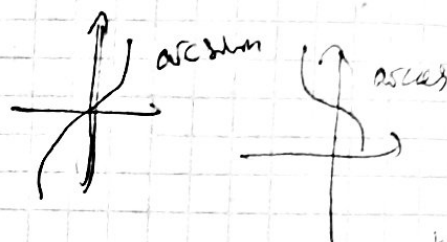
$$\sum_{n=1}^{\infty} a_n x^n, \quad \sum_{n=1}^{\infty} a_n = \lim_{x \rightarrow 1-0} \sum_{n=1}^{\infty} a_n x^n \Rightarrow \sum_{n=1}^{\infty} a_n = B(PA)$$

$$S = 1 + \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{4} \cdot \frac{1}{5} + \dots$$

$$\operatorname{arcsin} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \operatorname{arcsin} x = 1 + \sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n)!!} x^{2n-1}$$

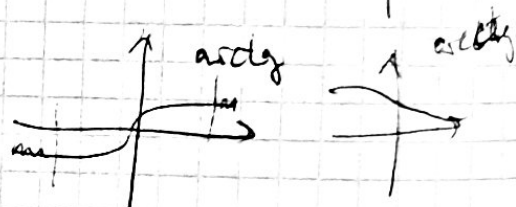
$$f(x) = \operatorname{arcsin} x = \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!} x^{2n+1} \quad (x)' = \frac{1}{\sqrt{1-x^2}}$$

$$S = \lim_{x \rightarrow 1-0} \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+1}}{2n+1} = \lim_{x \rightarrow 1-0} \operatorname{arcsin}(x) = \frac{\pi}{2}$$



Ex 3015

$$S = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \quad f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$



$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)} = \operatorname{arctg} x \quad \text{page 101}$$

$$\Rightarrow \text{Ex. } |x| < 1 \Rightarrow \lim_{x \rightarrow 1-0} \operatorname{arctg}(x) = \frac{\pi}{4}$$

Ex 3011

$$\sum_{n=1}^{\infty} n^2 x^{n-1} \text{ (a)}, \quad |x| < 1, \quad R=1 \quad \text{int. : } (-1, 1)$$

$$\sum_{n=1}^{\infty} n x^n = x \cdot \sum_{n=1}^{\infty} n x^{n-1} = x \cdot \sum_{n=1}^{\infty} (x^n)' = x \cdot \left(\sum_{n=1}^{\infty} x^n \right)' = x \cdot \left(\sum_{n=0}^{\infty} x^n - 1 \right)' = x \cdot \left(\frac{1}{1-x} \right)' \Rightarrow$$

$$\Rightarrow \text{Answer: } \frac{1+x}{(1-x)^2}$$

Ex 3014

$$S = 1 - \frac{1}{4} + \frac{1}{9} - \frac{2}{16} + \dots; \quad R=1, \quad (-1, 1);$$

$$f(x) = x - \frac{x^4}{4} + \frac{x^7}{7} - \frac{x^{10}}{10} + \dots$$

$$f'(x) = 1 - x^3 + x^6 - x^9 + \dots \quad \frac{1}{1+x^3} = f'(x) \quad \forall x < 1$$

$$\int_0^x \frac{1}{1+t^3} dt = \int_0^x \frac{1}{1+t^3} dt = \int_0^x \left(\frac{A}{1+t} + \frac{Bt+C}{1+t^2} \right) dt$$

Ap

Pr. prop. an: $\sin y = \frac{e^{iy} - e^{-iy}}{2i}, \cos y = \frac{e^{iy} + e^{-iy}}{2}$

(1) $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

(2) $\sin x + \cos y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

(3) $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$

(4) $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$

(1) $2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} = 2 \frac{1}{4i} (e^{i\frac{x+y}{2}} - e^{-i\frac{x+y}{2}}) (e^{i\frac{x-y}{2}} + e^{-i\frac{x-y}{2}}) = \frac{e^{i(x+y)} - e^{-i(x+y)}}{2i} =$

$= \frac{e^{ix}e^{iy} - e^{-ix}e^{-iy}}{2i} = \sin x + \sin y$

(2) $2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} = 2 \cdot \frac{1}{4i} (e^{i\frac{x+y}{2}} - e^{-i\frac{x+y}{2}}) (e^{i\frac{x-y}{2}} + e^{-i\frac{x-y}{2}}) = \frac{e^{ix}e^{iy} - e^{-ix}e^{-iy}}{2i} =$

$= \frac{1}{2i} (e^{i\frac{x+y}{2}} - e^{-i\frac{x+y}{2}}) (e^{i\frac{x-y}{2}} + e^{-i\frac{x-y}{2}}) = \frac{1}{2i} (e^{ix}e^{iy} - e^{-ix}e^{-iy}) = \sin x - \sin y$

Anhangung (3), (4)

$\sum_{k=1}^n \sin kx = \sum_{k=1}^n \frac{e^{ikx} - e^{-ikx}}{2i} = \frac{1}{2i} \frac{e^{ix} - e^{i(n+1)x}}{1 - e^{ix}} - \frac{1}{2i} \frac{e^{-ix} - e^{-i(n+1)x}}{1 - e^{-ix}} =$

$= \frac{1}{2i} \frac{1}{\sin \frac{x}{2}} \left(-e^{i\frac{x}{2}} + e^{i\frac{(n+1)x}{2}} - e^{-i\frac{x}{2}} + e^{-i\frac{(n+1)x}{2}} \right) = \frac{1}{2 \sin \frac{x}{2}} \left(-\frac{e^{ix}}{2} + \sin \left(n + \frac{1}{2} \right) x \right)$

301b

$1 - \frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \dots = \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)!!}{(2k)!!}$

$\frac{a_{n+1}}{a_n} = \frac{2(2n+1)!!}{(2n+2)!!} \cdot \frac{(2n)!!}{(2n-1)!!} =$

2 ne p. Leibniz m.

$\sum_{n=0}^{\infty} a_n = \lim_{x \rightarrow 1-0} \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)!!}{(2k)!!} x^k = \lim_{x \rightarrow 1-0} \left(1 + \sum_{k=1}^{\infty} \frac{(-1)^k (2k-1)!!}{(2k)!!} (-x) \right) = \frac{2n+1}{2n+2} < 1$

Starr m. (-1, 1)

$\frac{1}{\sqrt{1-x^2}} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \dots, 1 - \frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \dots = \lim_{x \rightarrow 0} \frac{1}{\sqrt{1-x^2}} = \frac{1}{2}$

301c

$S = \sum_{n=0}^{\infty} \frac{(2n+1)x^n}{n!}$

$R = \lim_{n \rightarrow \infty} \frac{a_n}{a_{n+1}} = \lim_{n \rightarrow \infty} \frac{2n+1}{n!} \cdot \frac{(n+1)!}{2n+3} = \lim_{n \rightarrow \infty} \frac{(n+1)(n+1)}{2n+3} = \infty$

(2) $S = \left(\sum_{n=0}^{\infty} \frac{(2n+1)x^n}{n!} \right)' = \left(\sum_{n=0}^{\infty} \frac{x^n}{n!} \right)' = \left(\frac{1}{x} e^{x^2} \right)' = \frac{1}{x} e^{x^2} \cdot 2x = 2e^{x^2}$

$$e^{x^2} \cdot (-1) \cdot \frac{1}{x^2} = e^{x^2} \left(2 - \frac{1}{x^2}\right).$$

$$\sqrt{3012} \quad \sum_{n=1}^{\infty} n(n+2)x^n = \left(\sum_{n=1}^{\infty} \frac{n(n+2)}{n+1} x^{n+1} \right)' = \left(\sum_{n=1}^{\infty} \frac{n}{n+1} x^{n+1} \right)' = \left(\sum_{n=1}^{\infty} \frac{1}{1+\frac{1}{n}} x^{n+1} \right)' =$$

$$\left(\frac{1}{x(1-x)} - \frac{1}{x} \left(\sum_{n=1}^{\infty} x^n \right) \right)' = \left(\frac{1}{x(1-x)} - \frac{1}{x} \left(\frac{x}{1-x} \right) \right)' = \left(\frac{1}{x(1-x)} - \frac{1}{1-x} \right)' = \left(\frac{1-x-x}{x(1-x)} \right)' = \left(\frac{-x}{x(1-x)} \right)' = \left(\frac{-1}{1-x} \right)' = \frac{1}{(1-x)^2}.$$

$$\sum_{n=1}^{\infty} n(n+2)x^n = x \left(\sum_{n=1}^{\infty} (n+2)x^n \right)' = x \left(\sum_{n=1}^{\infty} nx^n + 2 \sum_{n=1}^{\infty} x^n \right)' = x \left(\frac{3x-2x^2}{(1-x)^2} \right)' = \frac{x(3-x)}{(1-x)^3}.$$

$$\sqrt{3011} \quad \sum_{n=1}^{\infty} n^2 x^{n-1} = \frac{d}{dx} \left(\sum_{n=1}^{\infty} n x^n \right)' = \left(x \sum_{n=1}^{\infty} n x^{n-1} \right)' = \left(x \left(\sum_{n=1}^{\infty} x^n \right)' \right)' = \left(x \frac{(1-x) + x}{(1-x)^2} \right)' =$$

$$= \left(\frac{x}{(1-x)^2} \right)' = \frac{(1-x)^2 + 2x(1-x)}{(1-x)^4} = \frac{(1-x)(1-x+2x)}{(1-x)^4} = \frac{(1-x)(1+x)}{(1-x)^4} = \frac{1+x}{(1-x)^3}.$$

$$\sqrt{3007} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n}}{n(2n-1)} = \left(x \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{n(2n-1)} \right)' = x \left(\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-2}}{n} \right)' = x \int \frac{1}{x^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (x^2)^n}{n} = x \int \frac{1}{x^2} \ln(1+x^2) dx$$

$$= -\ln(1+x^2) - 2x \arctan x$$

$$\sqrt{3008} \quad \sum_{n=0}^{\infty} \frac{x^{4n+1}}{4n+1} = \int \left(\sum_{n=0}^{\infty} x^{4n} \right) dx = \int \frac{1}{1-x^4} dx = \int \frac{1+x}{(1+x^2)(1-x)(1+x)} dx$$

$$\frac{1}{(1+x^2)(1-x)(1+x)} \Leftrightarrow \frac{Ax+B}{(1+x^2)} + \frac{A_1}{1-x} + \frac{A_2}{1+x} \Rightarrow 1 = 2 \cdot 2 A_2, A_2 = A_3 = \frac{1}{4}$$

$$\Leftrightarrow \frac{A_1 x + A_2}{(1+x^2)} + \frac{4x + 4 - 2x}{(1-x)(1+x)} = \frac{A_1(x+1)}{(1+x^2)} + \frac{2}{(1-x)(1+x)}, \quad \text{Bsp. 1}$$

$$\int \frac{1}{(1+x^2)(1-x)(1+x)} = \frac{1}{4} \int \frac{1}{1-x} + \frac{1}{4} \int \frac{1}{1+x} +$$

$$= \frac{1}{4} \int \frac{x^2 x + 1}{1+x^2} dx = \frac{1}{4} \ln(1-x)(1+x) - \frac{1}{6} \left(\frac{1}{2} \ln(1+x^2) - 3 \arctan x \right) + 4 \left(2A_1 + \frac{1}{2} \right) + 1 + \frac{(1-x)}{3} = 1$$

$$\text{Also } \frac{1}{2} \int \frac{x}{1+x^2} dx = \frac{1}{4} \ln(1+x^2)$$

$$A_1 = \frac{1 - \frac{1}{3} - 2}{8} = \frac{-\frac{4}{3}}{8} = -\frac{1}{6}.$$

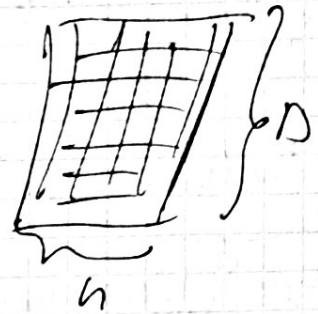
Kn. p. 569

$$f(x,y) \in C(\Pi)$$

$$\Pi = \{ a \leq x \leq b, c \leq y \leq d \}$$

$$T_{np} = \{ a = x_1 < x_2 < \dots < x_n = c, c = y_1 < y_2 < \dots < y_n = d \}$$

f-оп-на на Π .



$$\sum_{i=1}^n \sum_{k=1}^n f(\xi_i, \eta_k) \Delta x_i \Delta y_k - \text{upper-approximation}$$

$$(\xi_i, \eta_k) \in (x_{i-1}, x_i) \times (y_{k-1}, y_k], d = \max_{1 \leq i \leq n, 1 \leq k \leq n} \Delta x_i \Delta y_k$$

$$\lim_{d \rightarrow 0} S(f, T_{np}, \eta) = \int_a^b \int_c^d f(x,y) dx dy$$

$$T. \# f(x,y) \in C(\Pi) \Rightarrow \text{upper-approx}$$

Th. upper. $f(x,y)$ - op. univ. $\Leftrightarrow \forall \varepsilon > 0 \exists \delta : \delta - \delta < \varepsilon$.
Th. iter. $\exists \int_a^b \int_c^d f(x,y) dx dy = \int_a^b dx \int_c^d f(x,y) dy$, where $\exists \int_c^d f(x,y) dy \forall x \in [a,b]$.
 $= \int_c^d dy \int_a^b f(x,y) dx$

Ex 2906
 $\int_0^1 \int_0^1 (x+y) dx dy = \int_0^1 dx \int_0^1 (x+y) dy = \int_0^1 (xy + \frac{y^2}{2}) \Big|_0^1 dx = \int_0^1 (x + \frac{1}{2}) dx = (\frac{x^2}{2} + \frac{x}{2}) \Big|_0^1 = \frac{1}{2} + \frac{1}{2} = 1$.
 Аналогично аналогично.

Ex 2902
 $\int_0^1 \int_0^1 x y^2 dy = \int_0^1 (x \cdot \frac{y^3}{3}) \Big|_0^1 dx = \int_0^1 (\frac{x^4}{3} - \frac{x^2}{3}) dx = (\frac{x^5}{15} - \frac{x^3}{9}) \Big|_0^1 = \frac{1}{15} - \frac{1}{9} = \frac{1}{45}$.

$$\int_1^e dy \int_3^5 \frac{1}{y} dx = \int_1^e dy (\frac{x^5}{2y}) \Big|_3^5 = \int_1^e (\frac{25}{2y} - \frac{9}{2y}) dy = \int_1^e (\frac{25}{2} \ln|y| - \frac{9}{2} \ln|y|) \Big|_1^e = 8$$

$$\int_1^2 dx \int_0^1 \frac{dy}{(x+y+2)^2} = \int_1^2 dx \int_0^1 \frac{d(y+x+2)}{(x+y+2)^2} = \int_1^2 -\frac{1}{x+y+2} \Big|_0^1 dx = \int_1^2 (\frac{1}{x+3} - \frac{1}{x+2}) dx = \ln \frac{16}{15}$$

$$\int_0^1 dy \int_1^2 \frac{dx}{(x+y+2)^2} = \ln \frac{16}{15}$$

Ex 2925
 $\int_0^{\pi/2} d\varphi \int_0^a r \sin^2 \varphi dr = \int_0^{\pi/2} \sin^2 \varphi d\varphi (\frac{r^3}{3} \Big|_0^a) = \int_0^{\pi/2} \frac{a^3}{3} \sin^2 \varphi d\varphi = \frac{a^3}{3}$.

Ex 2911
 $(\int_a^b f(x) dx)^2 \leq (b-a) \int_a^b f^2(x) dx, (\int_a^b f(x) g(x) dx)^2 \leq \int_a^b f^2(x) dx \cdot \int_a^b g^2(x) dx$
 $\Leftrightarrow \int_a^b f^2(x) dx \int_a^b g^2(x) dx$

$$\int_a^b \int_a^b (f(x) - f(y))^2 dx dy = \int_a^b dx \int_a^b (f^2(x) - 2f(x)f(y) + f^2(y)) dy = \int_a^b f^2(x) dx - 2 \left(\int_a^b f(x) dx \right)^2$$

$$\int_a^b f^2(x) dx (b-a) \geq \left(\int_a^b f(x) dx \right)^2$$

$$\int_a^b \int_a^b (\lambda f(x) + g(y))^2 dx dy = \int_a^b (\lambda^2 f^2(x) + 2\lambda f(x)g(y) + g^2(y)) dx = \lambda^2 A + 2B\lambda + C \geq 0$$

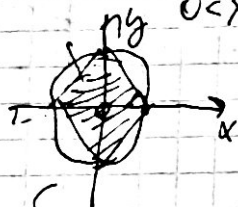
$$A = \int_a^b f^2(x) dx, B = \int_a^b f(x)g(y) dx, C = \int_a^b g^2(y) dy$$

$$(2B)^2 - 4AC \leq 0 \Rightarrow \left(\int_a^b f(x)g(y) dx \right)^2 \leq \int_a^b f^2(x) dx \int_a^b g^2(y) dy$$

2.9.12a)

$$\iint_{x^2+y^2 \leq 1} \ln(x^2+y^2) dx dy$$

$$0 < x^2+y^2 \leq 1, 1 \geq |x|+|y| \geq x^2+y^2, \ln(x^2+y^2) < 0 \Rightarrow \text{area} < 0$$



2.9.12b)

$$I = \iint_{x^2+y^2 \leq 1} \sqrt{1-x^2-y^2} dx dy$$

Рассмотрим три случая. $\Rightarrow$

$$I = \underbrace{\iint_{x^2+y^2 \leq 1} \sqrt{1-x^2-y^2} dx dy}_{>0} + \underbrace{\iint_{1 < x^2+y^2 \leq 2} \sqrt{1-x^2-y^2} dx dy}_{<0}$$

$$I > I_2 \Rightarrow$$

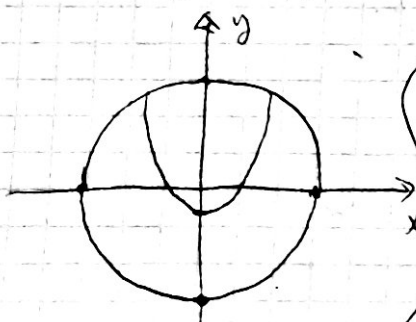
$$\iint_{1 < x^2+y^2 \leq 2} \sqrt{1-x^2-y^2} dx dy < 0$$

2/f

2.9.12b)

$$I = \iint_{x^2+y^2 \leq 4} \sqrt{1-(x^2+y^2)} dx dy, -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2} \Rightarrow I = \int_{-2}^2 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2} dy$$

$$x^2-y^2 \leq 1, y^2 \geq x^2-1$$



$$I = \iint_{0 \leq x \leq 1, -1 \leq y \leq 1-x} \operatorname{arcsinh}(x+y) dx dy = I_1 = \iint_{0 \leq x \leq 1, -1 \leq y \leq 0} \operatorname{arcsinh}(x+y) dx dy + \iint_{0 \leq x \leq 1, 0 \leq y \leq 1-x} \operatorname{arcsinh}(x+y) dx dy = I_1 + I_2, I_2 > 0$$

$$I_1 < 0, I_2 > 0$$

$$\text{Уменьш. } I_1 \text{ в } D = [0, 1] \times [-1, 0] \text{ } y = -x$$

$$f(x, y) = \operatorname{arcsinh}(x+y) \exists (y_1, y_2), (y_1', y_2') \text{ то}$$

$f(\xi_i, \eta_i) + f(\xi'_i, \eta'_i) = 0$; $\Pi = \{\bar{J}_i, i = \overline{1, n}\}$ т.е. Σ means every, every. for.
 $\cup \bar{J}_k$ wj. guar. D some $\epsilon > 0$. $\text{Lym} (\bar{\xi}_k, \bar{\eta}_k) \in \bar{J}_k$ $S_n(f) = \sum_k f(\bar{\xi}_k, \bar{\eta}_k) |\bar{J}_k|$
 $\text{Lym} \sup_{\text{over } \bar{J}_k} |f(x, y)| = M \Rightarrow |S_n(f)| \leq M \epsilon \Rightarrow \bar{I}_1 = \lim_{n \rightarrow \infty} S_n(f) = 0 \Rightarrow \bar{I} > 0$. ($\bar{I}$ is given as limit over Π)

§3213

$f(x, y) = \sin^2 x \sin^2 y$ в квадрате $0 \leq x \leq \pi, 0 \leq y \leq \pi$.

$$\bar{I} = \frac{1}{\pi^2} \int_0^\pi \int_0^\pi (\sin^2 x \sin^2 y) dx dy = \frac{1}{\pi^2} \int_0^\pi dx \int_0^\pi (\sin^2 x \sin^2 y) dy = \frac{1}{\pi^2} \int_0^\pi \frac{1}{2} \pi \sin^2 x dx$$

$$= \frac{1}{2\pi} \int_0^\pi \frac{1}{2} \pi dx = \frac{1}{4} (x)_0^\pi = \frac{\pi}{4}.$$

§3214

$$I = \iint_{x^2+y^2 \leq 100} \frac{dx dy}{100 + \cos^2 x + \cos^2 y}$$

I о функции $f(x, y)$ и given under domain D ,
 $2) g(x, y) \geq 0$ (≤ 0) every $\in D$

$$M = \sup_D f(x, y), m = \inf_D f(x, y) \text{ and}$$

$$\textcircled{1} \exists M \in \text{im } M : \iint_D f g dx dy = M \iint_D g dx dy$$

$$\text{even } f \text{ const. } \in D \Rightarrow f(\xi, \eta) : M = f(\xi, \eta)$$

$$I = \frac{200}{100 + \cos^2 \xi + \cos^2 \eta}$$

$$(\xi, \eta) : |\xi| + |\eta| \leq 10,$$

$$0 \leq \cos^2 \xi + \cos^2 \eta \leq 2,$$

$$100 < 100 + \cos^2 \xi + \cos^2 \eta < 102 \Rightarrow \frac{1}{102} < I < \frac{1}{100}.$$

§3215

$$\int_0^a dx \int_0^x f(x, y) dy = \int_0^a dy \int_y^a f(x, y) dx \quad (a > 0).$$

Пусть под. изображен T , пусть f const. на $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq a, 0 \leq y \leq x\}$
 $= \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq a, y \leq x \leq a\}$ или under region $\in D$

или на конформные члене мал. кубов, составлен охот.

$$F(x, y) = \begin{cases} f(x, y), & \text{если } (x, y) \in D \\ 0, & \text{если } (x, y) \in D_1 \setminus D \end{cases} \quad F : D_1 \rightarrow \mathbb{R}, D_1 = [0, a] \times [0, a].$$

$$\text{One } F \in \mathcal{F} : I_1 = \int_0^a dx \int_0^x f(x, y) dy, I_2 = \int_0^a dy \int_0^a F(x, y) dx.$$

Пусть-и разбиение Π квадрата D_1 на меньш-прямой $\bar{J}_i (i = \overline{1, n})$
 выберем произв. $(\xi_i, \eta_i) \in \bar{J}_i$ и составим:

$$S_n^{(1)}(F) = \sum_{i=1}^n \sum_{j=1}^n F(\xi_i, \eta_j) |\bar{J}_{ij}|, S_n^{(2)}(F) = \sum_{j=1}^n \sum_{i=1}^n F(\xi_i, \eta_j) |\bar{J}_{ij}|, |\bar{J}_{ij}| = \Delta x_i \Delta y_j$$

$$\text{We } \exists I_1, I_2 \Rightarrow \forall \epsilon > 0 \exists \delta > 0 : \forall \Pi \wedge d(\Pi) < \delta \Rightarrow |I_1 - S_n^{(1)}(F)| < \epsilon \text{ и}$$

$$|I_2 - S_n^{(2)}(F)| < \epsilon. \text{ Т.е. } S_n^{(1)}(F) = S_n^{(2)}(F) \Rightarrow I_1 = I_2. \text{ Пусть } x \in [0, a].$$

$$\text{Tezga } \int_0^a \int_0^x f(x,y) dy = \int_0^x \int_0^a f(x,y) dy + \int_x^a \int_0^x f(x,y) dy - \int_0^x \int_0^x f(x,y) dy$$

$$\text{Eben } y \in (0, a) \rightarrow \int_0^a f(x,y) dx = \int_0^x f(x,y) dx + \int_x^a f(x,y) dx - \int_0^x f(x,y) dx$$

$$\text{Nakazuy } \tilde{I}_1 = \tilde{I}_2, \text{ to } \int_0^a \int_0^x f(x,y) dy = \int_0^a dy \int_0^y f(x,y) dx$$

3.30.1

$$x = \frac{i}{n}, y = \frac{j}{n} \quad (i, j = 1, n-1); f(x,y) = xy, \quad \frac{1}{n^2} - \text{gen. k upab. berya.}$$

$$d(P) \rightarrow \text{ym } n \rightarrow \infty \Rightarrow \iint_{0 \leq x \leq 1, 0 \leq y \leq 1} xy \, dx \, dy = \lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{i=1}^n i \sum_{j=1}^n j = \lim_{n \rightarrow \infty} \frac{n(n+1)}{4n^2} = \frac{1}{4}$$

3.30.2

$$S_n(f), \bar{S}_n(f) \quad f(x,y) = x^2 y^2, (x,y) \in D, D = [1, 2] \times [1, 3] \quad x = 1 + \frac{i}{n}, y = 1 + \frac{j}{n} \\ \text{Nakazuy } \text{er.} = \frac{2}{n^2}; \quad (i, j = 1, n)$$

$$T_{ij} = \{(x,y) \in \mathbb{R}^2 : 1 + \frac{i}{n} \leq x \leq 1 + \frac{i+1}{n}, 1 + \frac{j}{n} \leq y \leq 1 + \frac{j+1}{n}\}$$

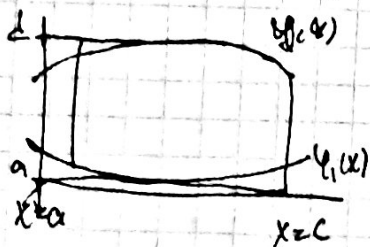
$$f_{\min} = f(1 + \frac{i}{n}, 1 + \frac{j}{n}), f_{\max} = f(1 + \frac{i+1}{n}, 1 + \frac{j+1}{n}) \quad \text{t.k. } f - \text{paccrova}$$

$$S_n(f) = \frac{2}{n^2} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} f(1 + \frac{i}{n}, 1 + \frac{j}{n}) = \frac{40}{3} - \frac{11}{n} + \frac{5}{3n^2}$$

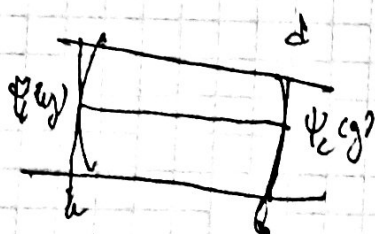
$$\bar{S}_n(f) = \frac{2}{n^2} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} f(1 + \frac{i+1}{n}, 1 + \frac{j+1}{n}) = \frac{40}{3} + \frac{11}{n} + \frac{5}{3n^2} \Rightarrow$$

$$\Rightarrow \lim_{n \rightarrow \infty} S_n(f) = \lim_{n \rightarrow \infty} \bar{S}_n(f) = 13 \frac{1}{3}$$

$$\int_a^b \int_c^d f(x,y) dx dy = \int_c^d dy \int_a^b f(x,y) dx \quad \text{Zn. pascro}$$

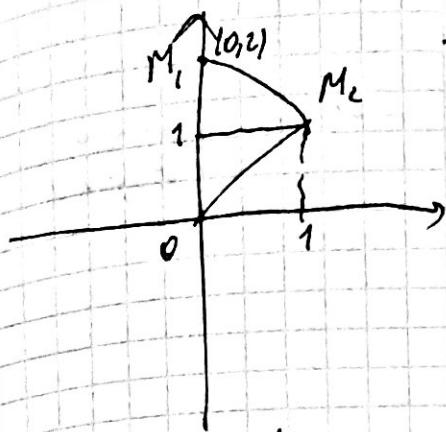


$$\int_a^b \int_c^d f(x,y) dx dy = \int_a^b dx \int_{y_1(x)}^{y_2(x)} f(x,y) dy \quad \text{-- ushuy no kurb. obrazu.}$$



$$\int_a^b \int_c^d f(x,y) dx dy = \int_c^d dy \int_{x_1(y)}^{x_2(y)} f(x,y) dx$$

$$I = \int_0^1 dx \int_x^{2-x^2} (x+y) dy$$



$$x=2-x^2, x^2+x-2=0 \Rightarrow x=-2, x=+1 \Rightarrow \int_{x=1}^1 y = -1$$

$$I = \int_0^2 dy \int_0^{\sqrt{2-y}} (x+y) dx + \int_0^1 dy \int_0^y (x+y) dx$$

$$\begin{aligned} I &= \int_0^2 dy \left(\frac{x^2}{2} + xy \right) \Big|_0^{\sqrt{2-y}} = \int_0^2 dy \left(\frac{2-y}{2} + \sqrt{2-y} y \right) = \\ &= \int_0^2 dy \left(y - \frac{1}{2} y^2 + \sqrt{2-y} y \right) \end{aligned}$$

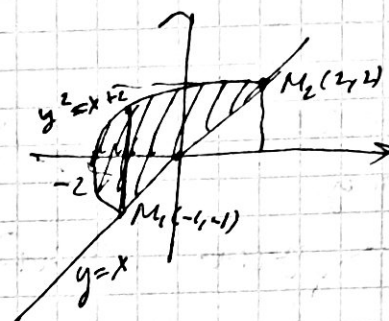
$$\int_0^2 \sqrt{2-y} y dy = \frac{1}{2} \int_0^2 \sqrt{2-y} dy^2 = \frac{1}{2} \left(\sqrt{2-y} y^2 + \int y^2 \cdot \frac{1}{2\sqrt{2-y}} dy \right)$$

$$\iint f(x,y) dx dy, \begin{cases} y^2 = x+2 \\ y = -2 \\ y = 0 \end{cases}$$

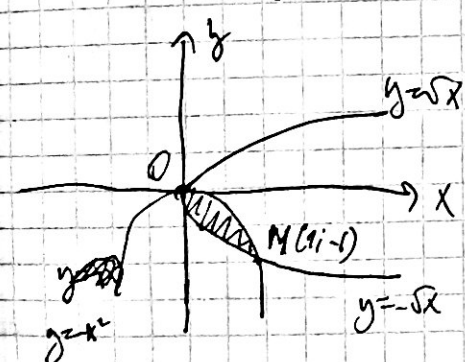
$f(x,y) \in C$

$$I = \int_{-1}^2 dy \int_{y^2-2}^y f(x,y) dx = \int_{-2}^{-1} dx \int_{-\sqrt{x+2}}^{\sqrt{x+2}} f(x,y) dy$$

$$\oplus \int_{-1}^2 dx \int_x^{x+2} f(x,y) dy$$



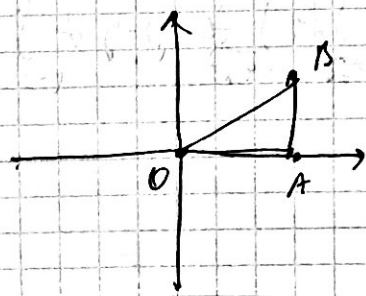
$$y^2 = x+2, \begin{cases} x = y^2 \\ y = -x^2 \end{cases}$$



$$\int_0^1 dx \int_{-x}^{-x^2} f(x,y) dy = \int_{-1}^0 dy \int_{y^2}^{\sqrt{-y}} f(x,y) dx$$

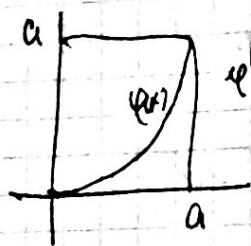
Σ 16

Σ 16: O(0,0), A(1,0), B(1,1)



$$\begin{aligned} \iint f(x,y) dx dy &= \int_0^1 dx \int_0^x f(x,y) dy = \\ &= \int_0^1 dy \int_y^1 f(x,y) dx \end{aligned}$$

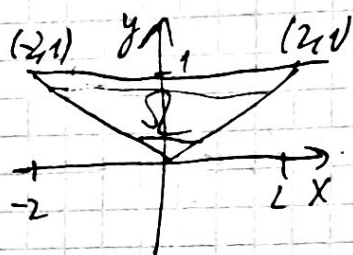
Формула Дирхле: $\int_0^a dx \int_0^x f(x,y) dy = \int_0^a dy \int_y^a f(x,y) dx$



$\varphi' > 0 \Rightarrow \int_0^a dx \int_0^{\varphi(x)} f(x,y) dy = \int_0^a dy \int_{\varphi^{-1}(y)}^a f(x,y) dx$ — обобщ. формула Дирхле

3912

$\Omega: O(0,0)$
 $A(2,1)$
 $B(-2,1)$

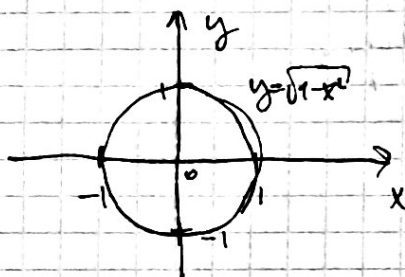


~~$\int dx \int f(x,y) dy$~~

$\int dy \int f(x,y) dx = \int_{-2}^0 dx \int_{-\frac{x}{2}}^1 f(x,y) dy +$

3918

$\Omega: x^2 + y^2 \leq 1$

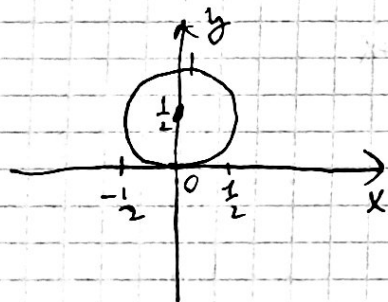


$\iint_{\Omega} f(x,y) dx dy = \int_{-1}^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y) dy = \int_{-1}^1 dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y) dx$

$y = \sqrt{1-x^2}$ $x = \pm \sqrt{1-y^2}$

$\Omega: x^2 + y^2 \leq y, \quad x^2 + y^2 - y = x^2 + y^2 - \frac{2}{2}y + \frac{1}{4} - \frac{1}{4} = x^2 + (y - \frac{1}{2})^2 - \frac{1}{4}$ $O(0; \frac{1}{2}),$
 $R = \frac{1}{2}.$

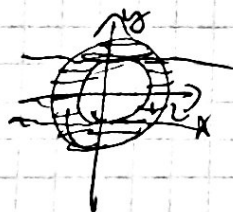
$\iint_{\Omega} f(x,y) dx dy = \int_{-\frac{1}{2}}^{\frac{1}{2}} dx \int_{-\sqrt{\frac{1}{4} - (y - \frac{1}{2})^2}}^{\sqrt{\frac{1}{4} - (y - \frac{1}{2})^2}} f(x,y) dy.$



3922

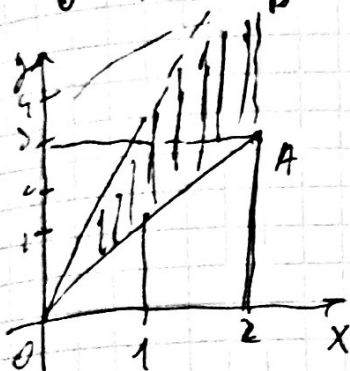
$\Omega: \begin{cases} x^2 + y^2 \geq 1 \\ x^2 + y^2 \leq 4 \end{cases}$

$\iint_{\Omega} f(x,y) dx dy = \int_{-2}^{-1} dx \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y) dy + \int_{-1}^1 dx \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y) dy + \int_{1}^2 dx \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y) dy$



3924

$$I = \int_0^c dx \int_x^{2x} f(x,y) dy, \quad \int_0^2 dy \int_{y/2}^y f(x,y) dx + \int_2^4 dy \int_{y/2}^2 f(x,y) dx$$



3926

$$I = \int_0^1 dx \int_{x^2}^{x^3} f(x,y) dy = \int_0^1 dy \int_{y^{1/3}}^{y^{1/2}} f(x,y) dx$$

$$x^3 \leq y \leq x^2, \quad 0 \leq y \leq 1$$

$$y^{1/3} \leq x \leq y^{1/2}, \quad 0 \leq x \leq 1$$

h/p 1) $h_2 > 0$ ок. P.K.D., unis. group.

2) A.A., A., Q., A.-A.; T. Myrman

3) P. ok., unis. ok. pabr.

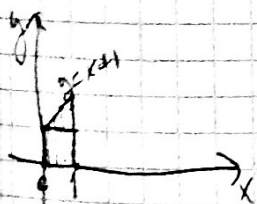
4) on ok., unis. on group. ok.; normen. group & unis. group.

5) pabr. - q. b. cimen. pabr.

6) A(A), A(C).

Q/p

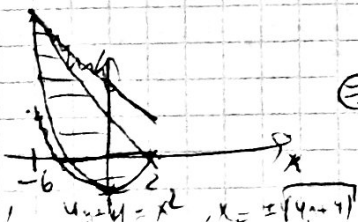
3918



$$\int_0^1 dx \int_0^{x+1} f(x,y) dy = \int_0^1 dy \int_0^1 f(x,y) dx + \int_1^2 dy \int_{y-1}^{y-1} f(x,y) dx$$

3925

$$\int_{-6}^2 dx \int_{x-1}^{2x} f(x,y) dy \Leftrightarrow$$

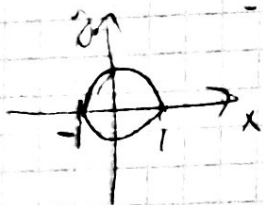


$$\Leftrightarrow \int_{-1}^0 dy \int_{-y-1}^{-y} f(x,y) dx + \int_0^2 dy \int_{y-1}^{y} f(x,y) dx$$

3917

$$\int_{-1}^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y) dy = \int_{-1}^1 dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y) dx + \int_0^1 dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y) dx$$

$-\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}$



483*

$$-\sqrt{1-x^2} = y, \quad x = \pm \sqrt{1-y^2} \quad | \quad 1-y^2 = y, \quad x = \pm \sqrt{1-y}$$

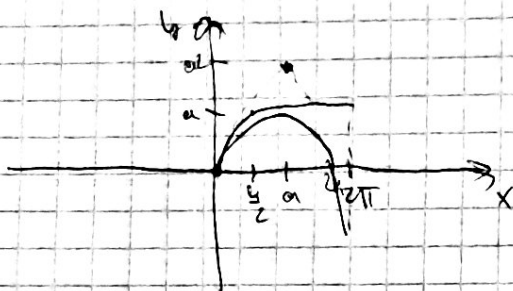
3929

$$\int_0^{2\pi} dx \int_{\sqrt{2a-x^2}}^{\sqrt{2a-x^2}} f(x,y) dy \quad (a>0) = \int$$

$$x_0 = \frac{-2a}{-2} = a \quad f(a) = 2a^2 - a^2 = a^2$$

$$x(2a-x)$$

$$\sqrt{2a^2 - 4a^2} = 2\sqrt{a^2 - a^2} = 2\sqrt{a-a}$$

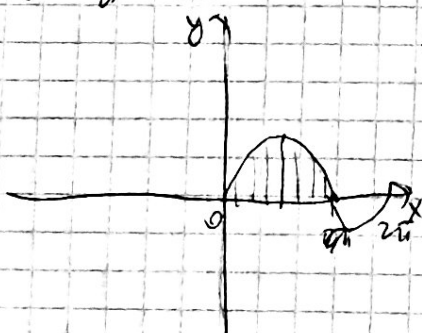


a) $a > \pi$
 b) $a \leq \pi$

$$\int_0^a dy \left(\int_{\frac{y}{2a}}^{\frac{a-\sqrt{a^2-y^2}}{2a}} f(x,y) dx + \int_{\frac{a+\sqrt{a^2-y^2}}{2a}}^{\frac{2a}{2a}} f(x,y) dx \right) + \int_a^{2a} dy \int_{\frac{y}{2a}}^{\frac{2a}{2a}} f(x,y) dx$$

3931

$$\int_0^{2\pi} dx \int_0^{\sin x} f(x,y) dy$$



~~483~~ x = arcsin y

$$\int_0^{2\pi} dx \int_{\sin x}^{\pi - \arcsin y} f(x,y) dx + \int_{-1}^0 dy \int_{\pi - \arcsin y}^{2\pi + \arcsin y} f(x,y) dx$$

211 (B)

81

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right)$$

$$S_n = \sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right) = \frac{1}{2} \left(1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{n} \right) - \left(\frac{1}{3} + \frac{1}{n} \right) - \frac{1}{n+1} \right)$$

$$= \frac{3}{2} + \frac{1}{n+1} - \frac{1}{n+2} \rightarrow \frac{3}{2}$$

$$\sqrt{2} \sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+3)}}$$

$$a_n = O^*\left(\frac{1}{k}\right) \quad n=1 \leq 1 - \text{pres.}$$

$$\sum_{k=1}^{\infty} \frac{\left(\frac{2k-1}{k+3}\right)^k}{a_k} = \sum_{k=1}^{\infty} \left(2 + \frac{-7}{k+3}\right)^k, \quad \frac{a_k}{a_{k+1}} = \left(\frac{2k-1}{k+3}\right)^k \cdot \frac{(k+4)^k}{(k+1)^k} \left(\frac{k+4}{2k+1}\right)^k =$$

$$= \left(\frac{(2k-1)(k+4)}{(k+3)2k+1}\right)^k \left(\frac{k+4}{2k+1}\right)^k, \quad a_k = \frac{2^k \left(k - \frac{1}{2}\right)^k}{(k+3)^k} =$$

$$\frac{2^k \left(1 + \frac{1}{k+3}\right)^k \left(1 - \frac{2}{2k+1}\right)^k}{e^{+ \frac{k}{k+3}} e^{-2 \frac{k}{2k+1}}} \cdot \frac{\frac{1}{2} \left(1 + \frac{1}{k}\right) \left(1 + \frac{2}{k}\right)^{-1}}{\frac{1}{2} \left(1 + \frac{1}{k}\right) \left(1 + \frac{2}{k}\right)^{-1}}$$

$$\frac{a_k}{a_{k+1}} = \left(1 + O\left(\frac{1}{k}\right)\right)^k \left(\frac{k+4}{2k+1}\right)^k \rightarrow$$

$$\rightarrow \frac{1}{2} + O\left(\frac{1}{k}\right) \Rightarrow \text{pres.}$$

$$\sqrt{4} \sum_{k=1}^{\infty} \frac{\sin \frac{\pi k}{3}}{k^\alpha + \sin \frac{\pi k}{3}} \quad | \text{asc. u. gen. exog.} |$$

$$\leq \left| \frac{\sin \frac{\pi k}{3}}{k^\alpha + \sin \frac{\pi k}{3}} \right| \leq \frac{1}{k^{\alpha + \sin \frac{\pi k}{3} - 1}} \quad \text{ex. } \alpha > 1$$

$$\frac{1}{2} \left(\frac{1}{k+3} \right)^{\frac{\pi k}{3}} \approx \frac{\cos \frac{\pi k}{3}}{k^{\alpha + \sin \frac{\pi k}{3}}}$$

pres. $\alpha \leq 1$

$$\frac{\sin \frac{\pi k}{3}}{k^\alpha} \left(1 + \frac{\sin \frac{\pi k}{3}}{k^\alpha}\right)^{-1} = \frac{\sin \frac{\pi k}{3}}{k^\alpha} \left(1 - \frac{\sin \frac{\pi k}{3}}{k^\alpha} + O\left(\frac{1}{k^\alpha}\right)\right) = \frac{\sin \frac{\pi k}{3}}{k^\alpha} - \frac{\sin^2 \frac{\pi k}{3}}{k^{2\alpha}} + O\left(\frac{1}{k^{2\alpha}}\right)$$

$$\sum_{k=1}^{\infty} \frac{\cos^2 \frac{\pi k}{3}}{k^{2\alpha}} \quad \alpha > 0 \text{ or } \alpha = 0, \text{ a } \sum_{n=1}^{\infty} \left(\frac{1}{k^{2\alpha}} + O\left(\frac{1}{k^{2\alpha}}\right)\right) \quad \text{ca. no } \sum, p > 0, \text{ it. u.}$$

$$\text{ca. } \frac{1}{\alpha} \geq \frac{1}{2}$$

$$\left| \sum_{k=1}^n \sin \frac{\pi k}{3} \right| < \frac{1}{\sin \frac{\pi}{6}}$$

$$\Rightarrow \text{ca. gen. } \frac{1}{2} \leq \alpha < 1, \text{ as. } \alpha > 1.$$

$$\frac{1}{2} \leq \alpha \leq 1$$

$$\sqrt{5} \text{ uen. } \text{for } p. \text{ ca.}$$

$$\sum_{k=1}^{\infty} \frac{x}{[k(k-1)x + 1] [kx + 1]}, \quad 0 < x < \infty \quad \int_n = \sum_{k=1}^n \frac{x}{[k(k-1)x + 1] [kx + 1]} \in$$

$$\textcircled{c} \sum_{k=1}^{\infty} \left(\frac{1}{(k-1)!} - \frac{1}{k!} \right) = 1 - \frac{1}{n+1}, \quad 0 < x < +\infty \quad \text{Series } \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k!} = 1$$

$$0 < \varepsilon_0 < \frac{1}{2}, \quad x = \frac{1}{n}; \quad \left| \beta_n\left(\frac{1}{n}\right) - \beta\left(\frac{1}{n}\right) \right| = \frac{1}{2} \Rightarrow \text{no factor.}$$

$$\Delta C$$

$$J = \sum_{k=1}^{\infty} (-1)^{k+1} x^k, \quad \frac{1}{x} \int \sum_{k=1}^{\infty} (-1)^{k+1} x^k dx = \frac{1}{x} \left(\sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^{k+1}}{k+1} + C_1 \right) = \frac{1}{x} \left(\frac{1}{x} \sum_{k=1}^{\infty} (-1)^{k+1} x^{k+1} + C_1 \right)$$

$$\textcircled{c} \frac{1}{x} \left(\frac{1}{x} (-\ln(1+x)) + C_1 \right) = -\frac{1}{x} \cdot \frac{x}{x^2} = -\frac{1}{x^2} = -\frac{x - (1+x)\ln(1+x)}{1+x}$$

$$J(x) = 0, \quad x = 0$$

$$\left(\sum_{k=1}^{\infty} (-1)^{k+1} x^k \right)' = x - x^2 + x^3 - \dots = -\frac{1}{1+x} + 1 = -\ln(1+x)$$

интеграл
двух переменных

$$\begin{cases} y_1 = \psi_1(x_1, x_2) \\ y_2 = \psi_2(x_1, x_2) \end{cases} \Rightarrow \begin{cases} x_1 = \psi_1^{-1}(y_1, y_2) \\ x_2 = \psi_2^{-1}(y_1, y_2) \end{cases} \quad \bar{y} = \psi(\bar{x}) \quad \frac{D(\bar{y})}{D(\bar{x})} \neq 0$$

$$\Rightarrow \iint_{\mathcal{D}} f(y_1, y_2) dy_1 dy_2 = \iint_{\mathcal{D}'} f(\psi_1(\bar{x}), \psi_2(\bar{x})) \cdot \left| \frac{D(\bar{y})}{D(\bar{x})} \right| d\bar{x} \quad (n=2)$$

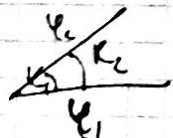
комп. групп. групп-а
D - open, связн., замкнутый
 $y \in \mathcal{D} = \bar{\mathcal{D}}, \quad x \in \mathcal{D}$

пример

$$y = Ax, \quad \begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \end{cases}, \quad \begin{cases} x'_r = \cos \varphi \\ x'_\varphi = -r \sin \varphi \end{cases}, \quad \begin{cases} y'_r = \sin \varphi \\ y'_\varphi = r \cos \varphi \end{cases}$$

$$\text{Итого } \iint_{\mathcal{D}} f(x, y) dx dy = \int_{\varphi_1}^{\varphi_2} \int_{r_1(\varphi)}^{r_2(\varphi)} f(r \cos \varphi, r \sin \varphi) r dr d\varphi$$

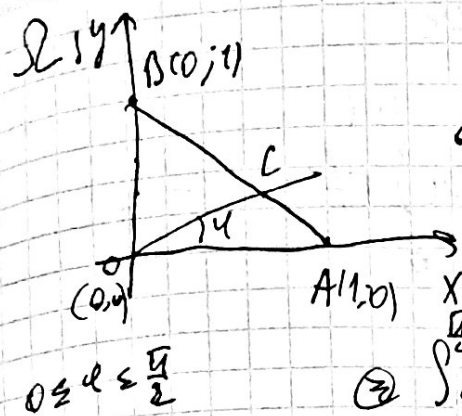
$$\frac{D(x, y)}{D(r, \varphi)} = \begin{vmatrix} \cos \varphi & -r \sin \varphi \\ \sin \varphi & r \cos \varphi \end{vmatrix} = r$$



↑ центр O на рис. D

формула: $\iint_{\mathcal{D}} f(x, y) dx dy = \int_{\varphi_1}^{\varphi_2} \int_{r_1(\varphi)}^{r_2(\varphi)} f(r \cos \varphi, r \sin \varphi) r dr d\varphi$

2.3.2
2.3.2.0



$\iint_R f(x,y) dx dy$; $xy=1$ $\cos \varphi + \sin \varphi = 1$

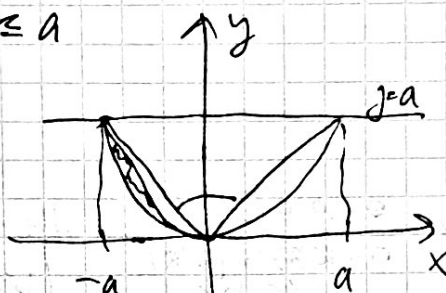
$R: \begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq 1-x \end{cases}$

$r(\varphi) = \frac{1}{\cos \varphi + \sin \varphi} = \frac{1}{\sqrt{2} \sin(\varphi + \frac{\pi}{4})}$

$\int_0^{\frac{\pi}{2}} d\varphi \int_0^{r(\varphi)} r f(r \cos \varphi, r \sin \varphi) dr$

2141

$R: \begin{cases} -a \leq x \leq a \\ \frac{x^2}{a} \leq y \leq a \end{cases}$



$y = a = 2a \sin^2 \varphi, \quad z = \frac{a}{\sin \varphi}$

$\frac{\pi}{4} \leq \varphi \leq \frac{3\pi}{4}$

$R = R_1 \cup R_2 \cup R_3$

$\int_0^{\frac{\pi}{4}} d\varphi \int_0^{\frac{a}{\cos \varphi}} r f(r \cos \varphi, r \sin \varphi) dr +$

$+ \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi \int_0^{\frac{a}{\sin \varphi}} r f(r \cos \varphi, r \sin \varphi) dr +$

$\int_{\frac{3\pi}{4}}^{\pi} d\varphi \int_0^{\frac{a}{\cos \varphi}} r f(r \cos \varphi, r \sin \varphi) dr$

$y = \frac{x^2}{2} = 2a \sin^2 \varphi$

$a \sin^2 \varphi \leq 2a \cos^2 \varphi$

$2 \cos^2 \varphi \leq \frac{a \sin^2 \varphi}{\cos^2 \varphi}$

$R_2: 0 \leq \varphi \leq \frac{\pi}{4}$

$R_3: \frac{3\pi}{4} \leq \varphi \leq \pi$

2145

$I = \int_0^2 dx \int_x^2 f(\sqrt{x^2+y^2}) dy$

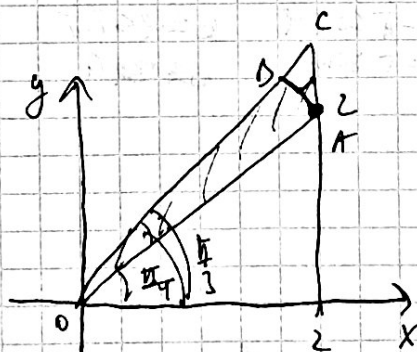
$x = r \cos \varphi$

$y = r \sin \varphi$

$r \cos \varphi = x, \quad r = \frac{x}{\cos \varphi}$

$r \sin \varphi = 2, \quad r = \frac{2}{\sin \varphi}$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_{\frac{x}{\cos \varphi}}^{\frac{2}{\sin \varphi}} r f(r) dr$



$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_0^2 r f(r) dr = \int_0^2 dr \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} r f(r) d\varphi$

$R_1: \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_{\frac{x}{\cos \varphi}}^{\frac{2}{\sin \varphi}} r f(r) dr$

2153

$\iint_R f\left(\frac{y}{x}\right) dx dy$

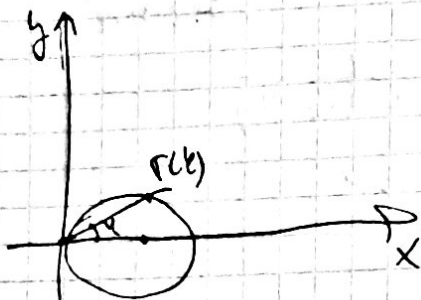
$x^2 + y^2 \leq x$

$$x = r \cos \varphi$$

$$y = r \sin \varphi$$

$$x^2 + y^2 \leq R, \quad r^2 \leq R \cos \varphi, \quad 0 \leq r \leq R \cos \varphi$$

$$(x - \frac{1}{2})^2 + y^2 \leq (\frac{1}{2})^2$$



$$\Omega: \left\{ -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2} \right\}$$

$$\iint_{\Omega} f(x, y) dx dy = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^{R \cos \varphi} r f(r \cos \varphi, r \sin \varphi) dr =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(r \cos \varphi) d\varphi \frac{R^2 \cos^2 \varphi}{2}$$

§ 3957

$$I = \int_a^b dx \int_{\alpha(x)}^{\beta(x)} f(x, y) dy$$

$$I = \int_a^b du \int_{\alpha(x)}^{\beta(x)} f(u, v) dv$$

$$0 < a < b$$

$$0 < \alpha(x) < \beta(x)$$

$$\alpha(x) < \beta(x)$$

$$u = x$$

$$v = \frac{y}{x}$$

$$x = u$$

$$y = xv = uv$$

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ v & u \end{vmatrix} = u$$

§ 3954

$$I = \iint_{x^2 + y^2 \leq a^2} \sqrt{x^2 + y^2} dx dy$$

$$x = r \cos \varphi \quad 0 \leq \varphi \leq 2\pi$$

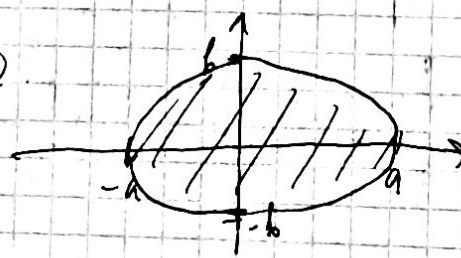
$$y = r \sin \varphi \quad 0 \leq r \leq a$$

$$I = \int_0^{2\pi} d\varphi \int_0^a r^2 dr = \int_0^{2\pi} 2\pi \frac{a^3}{3} d\varphi$$

§ 3962

$$I = \iint_{\Omega} \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}} dx dy$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$$



$$\frac{\partial(x, y)}{\partial(r, \varphi)} = \begin{vmatrix} a \cos \varphi & -a \sin \varphi \\ b \sin \varphi & b \cos \varphi \end{vmatrix} = ab$$

$$= ab r$$

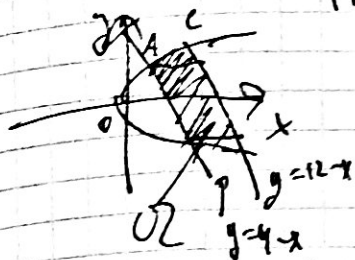
$$\textcircled{E} \left\{ \begin{array}{l} \text{abel. par. uersum. Koors.} \\ x = a r \cos \varphi \\ y = b r \sin \varphi, \quad 0 \leq \varphi \leq 2\pi; \quad 0 \leq r \leq 1 \end{array} \right.$$

$$\textcircled{E} \int_0^{2\pi} d\varphi \int_0^1 ab r \sqrt{1 - r^2} dr = \int_0^{2\pi} 2\pi ab \int_0^1 \sqrt{1 - r^2} dr = \frac{2}{3} \pi ab$$

§ 3969

$$\iint_D (x+y) dx dy$$

2



$$(4-x)^2 = 2x$$

$$x^2 - 10x + 16 = 0$$

$$D = 56$$

$$x_{1,2} = 2, 8$$

$$(2; 2); (8; -4)$$

$$(12-x)^2 = 2x$$

$$x_{1,2} = 8, 18$$

$$(8; 4); (18; 6)$$



$$\int_{-4}^4 dy \int_{\frac{y^2}{2}}^{12-y} (x+y) dx + \int_4^8 dy \int_{4-y}^{12-y} (x+y) dx + \int_8^{18} dy \int_{\frac{y^2}{2}}^{12-y} (x+y) dx \quad \textcircled{=}$$

$$\int (x+y) dx = \frac{x^2}{2} + xy + C; \quad \textcircled{=} \int_{-4}^4 \left(\frac{(12-y)^2}{2} + 12y - y^2 - \frac{y^4}{8} + \frac{y^3}{2} \right) dy + \dots \quad \textcircled{=}$$

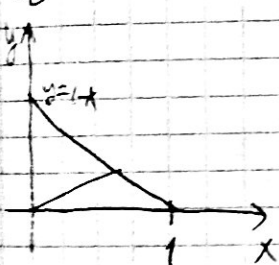
$$\textcircled{=} \left(72y - \frac{y^3}{3} + \frac{y^4}{8} \right) \Big|_{-4}^4 + \left(72y - \frac{y^3}{3} + \frac{y^4}{8} \right) \Big|_4^8 + \left(6y - \frac{y^3}{3} + \frac{y^4}{8} \right) \Big|_8^{18}$$

2/3

3940

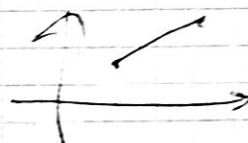
2-тр. $0 \leq x \leq 1; 0 \leq y \leq 1-x$,

$$\iint_D f(x,y) dx dy, \quad x = \dots; y = \dots$$



5-6 тр.

3942



прям.

3942

$$I = \iint_D f(x,y) dx dy; \quad D: (x^2+y^2)^2 = a^2(x^2-y^2) \quad (x \geq 0) \quad \text{ок. кр.}$$

$$x = r \cos \varphi$$

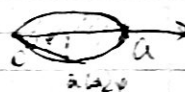
$$y = r \sin \varphi$$

$$a \cos 2\varphi$$

$$r^4 = a^2 r^2 \cos 2\varphi, \quad r^2 = a^2 \cos 2\varphi, \quad r = \pm a \cos 2\varphi$$

$$\cos 2\varphi \geq 0 \Rightarrow 2\varphi \in [0, \pi/2] \cup [3\pi/2, 2\pi]$$

$$I = \int_{-\pi/2}^{\pi/2} d\varphi \int_0^{a \cos 2\varphi} r \cdot f(r \cos \varphi, r \sin \varphi) dr$$



3550

$$\int_0^a d\varphi \int_0^a f(r) dr \quad (0 < \varphi < 2\pi)$$

$$0 \leq \varphi \leq a$$

$$0 \leq r \leq a$$

$$\int_0^a d\varphi \int_R^a f(r) dr$$

3355

$$\iint_{\pi^2 \leq x^2 + y^2 \leq 4\pi^2} \sin \sqrt{x^2 + y^2} \, dx \, dy = \int_0^{2\pi} d\theta \int_{\pi}^{2\pi} r \sin r \, dr = 2\pi \int_{\pi}^{2\pi} r \sin r \, dr = -6\pi^2$$

$$\pi^2 \leq x^2 + y^2 \leq 4\pi^2$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\pi \leq r \leq 2\pi$$

$$\frac{1}{2} \left(\int_{\pi}^{2\pi} r^2 \sin r \, dr \right) = \frac{1}{2} (r^2 \sin r - \int r^2 \cos r \, dr)$$

$$\frac{1}{2} \left(\int_{\pi}^{2\pi} r^2 \cos r \, dr \right) = \left| \begin{matrix} r \sin r = t \\ r = \arcsin t \\ dr = \frac{1}{\sqrt{1-t^2}} dt \end{matrix} \right|$$

$$= \int \frac{t \arcsin t}{\sqrt{1-t^2}} dt$$



3358

$$\int_0^2 \int_{1-x}^{2-x} f(x,y) \, dy \, dx$$

$$\frac{\partial x}{\partial u} \frac{\partial x}{\partial v}$$

$$\frac{\partial y}{\partial u} \frac{\partial y}{\partial v}$$

$$\begin{aligned} u &= x+y \\ v &= x-y \\ du &= dx+dy \\ dv &= dx-dy \\ 1 &= \frac{dx}{dv} + \frac{dy}{dv} \end{aligned}$$

$$\begin{aligned} \frac{\partial x}{\partial u} &= \frac{1}{2}, \quad \frac{\partial x}{\partial v} = \frac{1}{2} \\ \frac{\partial y}{\partial u} &= \frac{1}{2}, \quad \frac{\partial y}{\partial v} = -\frac{1}{2} \end{aligned}$$

$$\frac{\partial x}{\partial u} = \frac{1}{2}, \quad \frac{\partial x}{\partial v} = \frac{1}{2}$$

$$\frac{\partial y}{\partial u} = \frac{1}{2}, \quad \frac{\partial y}{\partial v} = -\frac{1}{2}$$

$$0 \leq x \leq 2, 0 \leq \frac{y+v}{2} \leq 2$$

$$1-x \leq y \leq 2-x$$

$$\frac{2-v}{2} \leq \frac{u+v}{2} \leq \frac{4-u}{2}$$

$$1 \leq u \leq 3$$

$$u \leq v \leq 4-u$$

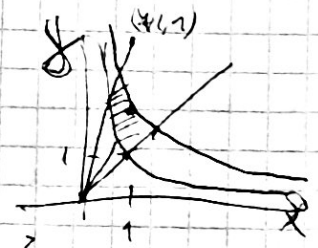
$$1 \leq u \leq 3$$

$$\frac{1}{4} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -\frac{1}{2}$$

$$\frac{\partial x}{\partial v} \frac{\partial x}{\partial u}$$

3364

$$\iint_{\mathcal{R}} f(x,y) \, dx \, dy, \quad \mathcal{R}: \begin{cases} xy=1 \\ xy=2 \\ y=x \\ y=4x \end{cases} \quad (x>0, y>0)$$



$$xy=4, \quad \frac{\partial x}{\partial u} \frac{\partial x}{\partial v}$$

$$y = \frac{1}{x}, \quad x^2 = \frac{1}{y}, \quad x = \frac{1}{\sqrt{y}}$$

$$x = \frac{1}{\sqrt{y}}$$

$$x = \frac{2}{y}$$

$$x = \frac{1}{\sqrt{y}}$$

$$x = \frac{2}{y}$$

$$\frac{1}{2} \leq x \leq \sqrt{2}$$

$$1 \leq y \leq \infty$$

$$\frac{1}{2} = \int_{\pi/4}^{\pi/2} \int_{\frac{1}{2} \sec 2\theta}^{\sec 2\theta} r f(\frac{1}{2} r \cos 2\theta) \, dr \, d\theta$$

$$x = r \cos \theta, \quad y = r \sin \theta$$

$$u = xy, \quad v = \frac{y}{x}, \quad 1 \leq u \leq 2, \quad 1 \leq v \leq 4, \quad |J| = \frac{1}{2v}$$

$$\iint_{\mathcal{R}} f(x,y) \, dx \, dy = \int_1^2 \int_1^4 f(u) \, du \, dv = \ln 2 \cdot \int_1^4 f(u) \, du$$

$$\begin{aligned} \frac{\partial x}{\partial u} \frac{\partial x}{\partial v} &= \frac{2}{r^2 \sin 2\theta} \\ \frac{\partial y}{\partial u} \frac{\partial y}{\partial v} &= \frac{1}{r^2 \sin 2\theta} \\ \frac{1}{2} r^2 \sin 2\theta &= 2 \\ r &= \frac{2}{\sin 2\theta} \end{aligned}$$

$\int_{-\pi/2}^{\pi/2} \int_0^1 r^2 dr d\varphi = \int_{-\pi/2}^{\pi/2} d\varphi \int_0^1 r^2 dr = \int_{-\pi/2}^{\pi/2} \left[\frac{r^3}{3} \right]_0^1 d\varphi = \frac{1}{3} \int_{-\pi/2}^{\pi/2} d\varphi = \frac{1}{3} \cdot \pi = \frac{\pi}{3}$
 $x = r \cos \varphi, y = r \sin \varphi, r \leq \sqrt{1-x^2}$
 $\frac{2x}{2r} = \frac{1}{r} \cos \varphi = \frac{1}{\sqrt{1-x^2}} \cos \varphi$
 $\frac{dy}{dr} = \frac{1}{2} \frac{1}{r} \frac{2x}{2r} = \frac{1}{2} \frac{1}{r} \frac{1}{\sqrt{1-x^2}} \cos \varphi$

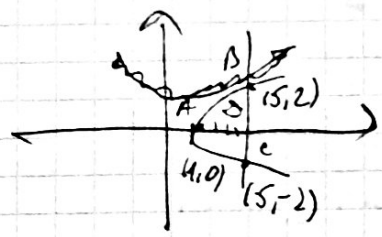
$\frac{\partial f(x,y)}{\partial r} = \frac{1}{r} (\sqrt{1-x^2} \cos \varphi + \sqrt{1-x^2} \sin \varphi)$
 $\frac{1}{\sqrt{1-x^2}} = \frac{\cos \varphi + \sin \varphi}{\sqrt{1-x^2}} = \frac{\cos \varphi + \frac{2x}{2r}}{\sqrt{1-x^2}} = \frac{\cos \varphi}{\sqrt{1-x^2}} + \frac{1}{r}$

$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-r^2 \cos^2 \varphi}}$
 $I = \int_0^1 \int_0^{\pi/2} \frac{1}{\sqrt{1-r^2 \cos^2 \varphi}} r^2 dr d\varphi = 2 \int_0^{\pi/2} d\varphi \int_0^1 \frac{r^2}{\sqrt{1-r^2 \cos^2 \varphi}} dr = 2 \int_0^{\pi/2} \frac{1}{\cos^2 \varphi} d\varphi = 2 \int_0^{\pi/2} \sec^2 \varphi d\varphi = 2 \tan \varphi \Big|_0^{\pi/2} = \pi$
 $= 2 \int_0^1 \frac{1+t^2}{1-t^2} dt = \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{t-1}{t+1} \Big|_0^1 = \frac{\pi}{2}$

Кр. задача

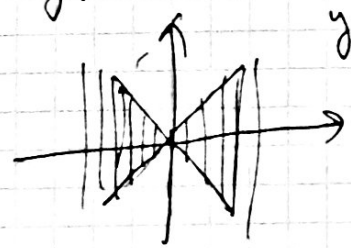
0-огр. замкну. область; $|D| = \iint_D |D(x,y)| dx dy$

$D: \begin{cases} x = y^2 + 1 \\ x = 5 \end{cases}$



$|D| = \int_1^5 dx \int_{-\sqrt{x-1}}^{\sqrt{x-1}} dy = \int_1^5 2\sqrt{x-1} dx = 2 \frac{(x-1)^{3/2}}{3/2} \Big|_1^5 = \frac{4}{3} (4^{3/2}) = \frac{4}{3} \cdot 8 = \frac{32}{3}$

$E: (x^2 + y^2)^2 = 2a^2(x^2 - y^2) \geq 0$
 $x^2 \geq y^2, |x| \geq |y|$



$x = r \cos \varphi, y = r \sin \varphi$

$r^4 = 2a^2 r^2 \cos 2\varphi$
 $r^2 = 2a^2 \cos 2\varphi \geq 0, \varphi \in [-\pi/4, \pi/4]$
 $r = \sqrt{2} a \sqrt{\cos 2\varphi}$

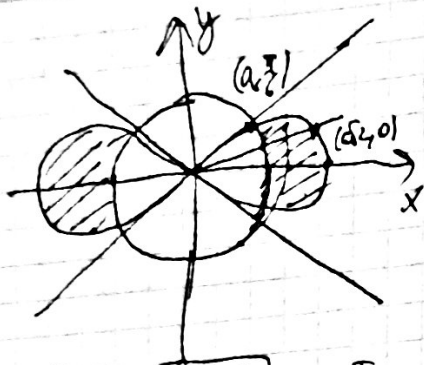
Уз зертхана $\Rightarrow$ ному мезгил мезг. $\frac{1}{4}$ жана

$|D| = \int_0^{\pi/4} \int_0^{\sqrt{2} a \sqrt{\cos 2\varphi}} r^2 dr d\varphi = \int_0^{\pi/4} \frac{r^3}{3} \Big|_0^{\sqrt{2} a \sqrt{\cos 2\varphi}} d\varphi = \int_0^{\pi/4} \frac{2a^3 \cos 2\varphi}{3} d\varphi = \frac{a^3}{3} \sin 2\varphi \Big|_0^{\pi/4} = \frac{a^3}{6}$

$|D| = 2a^2$

Задача 3.5.87

$D: (x^2 + y^2)^2 = 2a^2(x^2 - y^2)$
 $x^2 + y^2 \geq a^2$



by using polar coordinates to find the area of the shaded region

$$r^2 = a^2 \cos 2\phi$$

$$\frac{r^2}{2} = \cos 2\phi$$

$$r^2 = a^2 \cos 2\phi$$

$$\cos 2\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{6}$$

$$A = \int_0^{\pi/6} \int_0^{a\sqrt{\cos 2\phi}} r dr d\phi = \int_0^{\pi/6} \left(\frac{a^2 \cos 2\phi}{2} - \frac{a^2}{2} \right) d\phi = \frac{\sqrt{3}a^2}{4} - \frac{\pi a^2}{8} = \frac{(3\sqrt{3} - \pi)a^2}{8}$$

Σ 1989

$$(x^2 + y^2)^2 = a(x^3 - 3xy^2) \geq 0 \quad x = r \cos \phi, \quad y = r \sin \phi$$

$$r^4 = 2r^3 \cos \phi \int_0^{\pi/2} r dr d\phi$$

$$+ 2 \int_0^{\pi/2} \int_0^{a \cos \phi} r dr d\phi$$

$$A = \int_0^{\pi/2} \cos^3 \phi d\phi = \int_0^{\pi/2} \left(\frac{1 + \cos 2\phi}{2} \right) d\phi$$

$$A = \cos^3 \phi - \frac{1}{2} \cos \phi$$

$$\int_0^{\pi/2} \cos^3 \phi d\phi$$

$$\cos^3 \phi = \cos \phi (1 - \sin^2 \phi)$$

$$\int_0^{\pi/2} \cos \phi (1 - \sin^2 \phi) d\phi$$

$$\cos^2 \phi (1 - \sin^2 \phi) = \frac{1}{3} (1 - \cos^2 \phi) d\cos \phi$$

$$\cos^3 \phi - \cos \phi$$

$$\frac{1 + \cos 2\phi}{2}$$

Σ 1989

Σ 1989

$$\begin{aligned} x+y &= a \\ x+y &= b \\ a &= 2x \\ b &= 2y \end{aligned} \quad 0 < a < b, \quad 0 < x < y$$

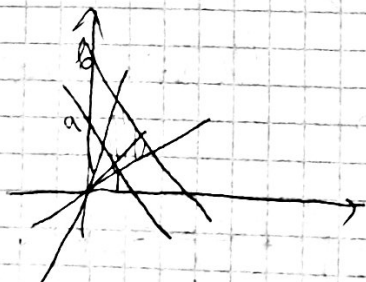
$$x + 2x = a \Rightarrow x = \frac{a}{3}$$

$$x = \frac{a}{3}$$

$$y = \frac{2a}{3}$$

$$\int_0^{\pi/2} \int_0^{a \cos \phi} r dr d\phi$$

$$\int_0^{\pi/2} \cos^3 \phi d\phi$$



$$x = r \cos \phi, \quad y = r \sin \phi, \quad a \leq r \cos \phi + r \sin \phi \leq b$$

$$a \leq r \sqrt{2} \sin(\phi + \frac{\pi}{4}) \leq b$$

$$\frac{a}{\sqrt{2} \sin(\frac{\pi}{4} + \phi)} \leq r \leq \frac{b}{\sqrt{2} \sin(\frac{\pi}{4} + \phi)}$$

$$\arcsin \frac{a}{\sqrt{2}} \leq \phi \leq \arcsin \frac{b}{\sqrt{2}}$$

$$\log a = \frac{2a}{3} \cdot \frac{1}{a} = \frac{2}{3}$$

$$\int_0^{\pi/2} \int_0^{a \cos \phi} r dr d\phi = \frac{1}{2} \int_0^{\pi/2} \cos^3 \phi d\phi = \frac{1}{2} \left(\frac{1}{3} \cos^3 \phi - \cos \phi \right) \Big|_0^{\pi/2} = \frac{1}{6}$$

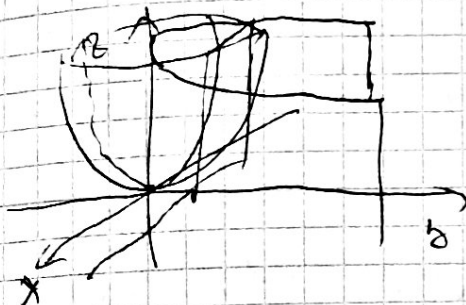
$$\textcircled{=} \frac{a^2 - b^2}{4} \int_{\arccos \frac{a}{b}}^{\arccos \frac{a}{b}} \frac{1}{\sin^2(\frac{\pi}{4} + x)} = \frac{a^2 - b^2}{4} (-\cot(\frac{\pi}{4} + x)) \Big|_{\arccos \frac{a}{b}}^{\arccos \frac{a}{b}} = \frac{a^2 - b^2}{4} \left(\frac{1}{\tan(\arccos \frac{a}{b} + x)} \right) \Big|_{\arccos \frac{a}{b}}^{\arccos \frac{a}{b}}$$

$$\therefore \frac{a^2 - b^2}{4} \left(\frac{1}{\tan(\pi + \arccos \frac{a}{b})} \right); \quad x+y=u, \quad \frac{y}{x}=v, \quad a \leq u \leq b, \quad \alpha \leq v \leq \beta, \quad |J| = \frac{u}{(1+v)^2}$$

$$\int = \int_a^b u du \int_{\alpha}^{\beta} \frac{dv}{(1+v)^2} = \frac{1}{2} \frac{(b-a)(b^2-a^2)}{(1+\alpha)(1+\beta)}$$

4003

$$z = x^2 + y^2, \quad y = x^2, \quad y = 1, \quad z = 0$$

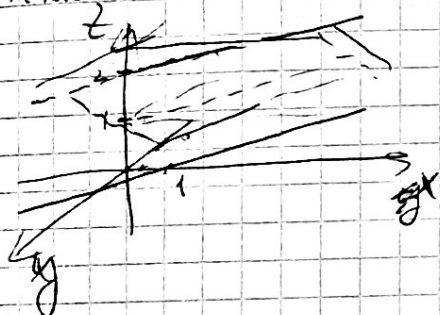


$$V = \int_{-1}^1 dx \int_{x^2}^1 (x^2 + y^2) dy = \frac{88}{105}$$

4002

$$z = 1 + x + y, \quad z \geq 0, \quad x + y = 1, \quad x = 0, \quad y = 0$$

$$y = 1 - x + z$$



$$V = \int_0^1 dx \int_0^{1-x} (1 + x + y) dy = \int_0^1 \left(\frac{y}{2} - x + \frac{x^2}{2} \right) dx = \frac{5}{6}$$

4011

$$z = xy, \quad x + y + z = 1, \quad z = 0$$

$$y = \frac{1-x}{1+x}$$

$$\iint xy \cdot z = xy \cap \{x+y+z=1\} \text{ no constraints}$$

$$\text{on } xy: y = \frac{1-x}{1+x}, \quad x \in \mathbb{R} \setminus \{-1\}$$

$$\Rightarrow D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 1-x\}$$

$$= D_1 = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq \frac{1-x}{1+x}\} \cup$$

$$D_2 = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, \frac{1-x}{1+x} \leq y \leq 1-x\}$$

$$\text{on } D_1: f(x, y) = xy, \quad \text{on } D_2: f(x, y) = 1 - (x + y)$$

$$V = \iint_D f(x, y) dx dy = \int_0^1 dx \int_0^{\frac{1-x}{1+x}} y dy + \int_0^1 dx \int_{\frac{1-x}{1+x}}^{1-x} (1-x-y) dy = \frac{1}{2} \int_0^1 \frac{x(1-x)^2}{1+x} dx =$$

$$= \frac{1}{2} \int_0^1 \left(x^2 - 3x + 4 - \frac{4}{1+x} \right) dx = \frac{1}{2} - 2 \ln 2$$

3914

$$z = xy, (x^2 + y^2)^2 = 2xy, z \geq 0 (x > 0, y > 0)$$

3915

$$x = r \cos \varphi, y = r \sin \varphi; r^2 = 2 \sin \varphi \cos \varphi = \sin 2\varphi, z = r(\cos \varphi + \sin \varphi)$$

$$V = \iiint_V (x^2 + y^2) dx dy dz = \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{\sin 2\varphi}} \int_0^{\sin 2\varphi} r^2 (\cos \varphi + \sin \varphi) r dr d\varphi = \frac{2\sqrt{2}}{5} \int_0^{\frac{\pi}{2}} (\sin^{\frac{5}{2}} \varphi \cos^3 \varphi + \sin^{\frac{5}{2}} \varphi \sin^3 \varphi) d\varphi$$

$$= \frac{\sqrt{2}}{6} \frac{\pi}{\sin \frac{\pi}{4}} = \frac{\pi}{8}$$

3955

$$\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1, x=0, y=0$$

$$x = a \cos^2 \varphi, y = b \sin^2 \varphi (0 \leq \varphi \leq \frac{\pi}{2})$$

$$|J| = 8ab \cos^2 \varphi \sin^2 \varphi$$

$$\sqrt{\frac{1}{a} \sin^2 \varphi + \frac{1}{b} \cos^2 \varphi} = 1, \varphi = \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$$

$$\varphi = \pi k, k \in \mathbb{Z}$$

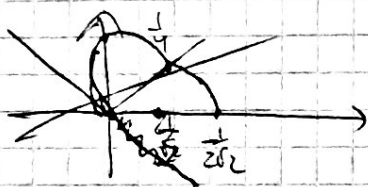
$$r = \frac{1}{\sqrt{\frac{1}{a} \sin^2 \varphi + \frac{1}{b} \cos^2 \varphi}} (0 \leq \varphi \leq \frac{\pi}{2})$$

$$\varphi = \pi k, k \in \mathbb{Z}$$

$$S = \iint_S 8ab \cos^2 \varphi \sin^2 \varphi d\varphi = 4ab \int_0^{\frac{\pi}{2}} \cos^2 \varphi \sin^2 \varphi d\varphi =$$

$$= 4ab \int_0^1 u^2 (1-u^2)^2 du = 4ab \int_0^1 (u^2 - 2u^4 + u^6) du =$$

$$= 4ab \left(\frac{1}{3} - \frac{2}{5} + \frac{1}{7} \right) = \frac{16}{70}$$



$$u = \cos \varphi, du = -\sin \varphi d\varphi, d\varphi = -\frac{1}{\sin \varphi} du$$

3903

$$x^2 + y^2 + z^2 - xy - xz - yz = a^2 \text{ or } x + y + z = 0,$$

$$(x+y+z)^2 - 3(xy+xz+yz) = a^2,$$

$$= 0$$

$$-3(xy+x(-x-y)+y(-x-y)) = a^2$$

$$-3(-x^2-y^2-xy) = a^2,$$

$$x^2 + y^2 + xy = \frac{a^2}{3}$$

$$x' = \frac{1}{\sqrt{2}} x - \frac{1}{\sqrt{2}} z,$$

$$y' = \frac{1}{\sqrt{6}} x - \frac{2}{\sqrt{6}} y + \frac{1}{\sqrt{6}} z$$

$$z' = \frac{1}{\sqrt{3}} x + \frac{1}{\sqrt{3}} y + \frac{1}{\sqrt{3}} z,$$

$$z' = \frac{1}{\sqrt{3}} (x+y+z) = \frac{b}{\sqrt{3}} = 0$$

$$x = \frac{1}{\sqrt{2}} x' + \frac{1}{\sqrt{6}} y' + \frac{1}{\sqrt{3}} z'$$

$$y = -\frac{1}{\sqrt{6}} y' + \frac{1}{\sqrt{3}} z'$$

$$z = -\frac{1}{\sqrt{2}} x' + \frac{1}{\sqrt{6}} y' + \frac{1}{\sqrt{3}} z'$$

$$\Rightarrow \frac{5}{2} (x'^2 + y'^2),$$

$$x'^2 + y'^2 = \frac{2}{5} a^2,$$

$$S = \iint_{x'^2 + y'^2 \leq \frac{2}{5} a^2} dx' dy' = \frac{2}{5} \pi a^2$$

3975

$$(x^3 + y^3)^2 = x^2 + y^2, x \geq 0, y \geq 0,$$

$$r^2 = \frac{1}{\cos^3 \varphi + \sin^3 \varphi} (0 \leq \varphi \leq \frac{\pi}{2})$$

$$S = \int_S r dr d\varphi = \frac{1}{2} \int_0^{\pi} r^2 d\varphi = \frac{1}{2} \int_0^{\pi} \frac{d\varphi}{\cos^2 \varphi + \sin^2 \varphi} = \frac{1}{2} \int_0^{\pi} \frac{d\varphi}{1} = \frac{1}{2} \left(\frac{2}{\sin \varphi \cos \varphi} + \frac{\sin \varphi \cos \varphi}{1 - \sin \varphi \cos \varphi} \right)$$

$$\int \frac{d\varphi}{\sin \varphi \cos \varphi} = \frac{1}{\sin \varphi} \int \frac{d\varphi}{\cos \varphi} = \frac{1}{\sin \varphi} \ln \left| \frac{1 + \sin \varphi}{1 - \sin \varphi} \right| = \frac{1}{\sin \varphi} \ln \left| \frac{1 + \sin \varphi}{1 - \sin \varphi} \right|$$

$$-\ln \left| \frac{1 - \sin \varphi}{1 + \sin \varphi} \right| = \ln(1 + \sin \varphi), \quad \int \frac{\sin \varphi + \cos \varphi}{1 - \sin \varphi \cos \varphi} d\varphi = 2 \int \frac{d(\frac{1}{2} \sin \varphi - \frac{1}{2} \cos \varphi)}{2(\frac{1}{2} \sin \varphi - \frac{1}{2} \cos \varphi)^2 + \frac{1}{2}} =$$

$$\frac{2}{\sqrt{2}} \ln(1 + \sin \varphi) \Big|_0^{\pi/4} = \pi, \quad \int = \frac{\sqrt{2}}{2} \ln(1 + \sin \varphi) + \frac{\pi}{6}$$

10.3.3.4

$$a \left(\frac{1}{h^2} + \frac{1}{k^2} \right) = \frac{x^2}{h^2} + \frac{y^2}{k^2} \quad (x > 0, y > 0), \quad x = a \cos \varphi, \quad y = b \sin \varphi$$

$$r^2 = \frac{(a/h)^2 \cos^2 \varphi + (b/k)^2 \sin^2 \varphi}{(\cos^2 \varphi + \sin^2 \varphi)^2}, \quad \left(\frac{a}{h} \right)^2 \cos^2 \varphi - \left(\frac{b}{k} \right)^2 \sin^2 \varphi \geq 0, \quad \left(\frac{ak}{bh} \right)^2 \geq \tan^2 \varphi, \quad 0 \leq \varphi \leq \arctan \frac{ak}{bh}$$

$$S = \iint_S ab r dr d\varphi = \frac{ab}{2} \int_0^{\arctan \frac{ak}{bh}} r^2 d\varphi = \frac{ab}{2} \left(\frac{a}{h} \right)^2 \int_0^{\arctan \frac{ak}{bh}} \frac{\cos^2 \varphi d\varphi}{(\cos^2 \varphi + \sin^2 \varphi)^2} - \frac{ab}{2} \left(\frac{b}{k} \right)^2 \int_0^{\arctan \frac{ak}{bh}} \frac{\sin^2 \varphi d\varphi}{(\cos^2 \varphi + \sin^2 \varphi)^2}$$

$$= \frac{ab}{2} \left(\frac{a}{h} \right)^2 \left(-\frac{1}{1 + \tan^2 \varphi} \right) \Big|_0^{\arctan \frac{ak}{bh}} - \frac{ab}{2} \left(\frac{b}{k} \right)^2 \left(\frac{1}{1 + \tan^2 \varphi} \right) \Big|_0^{\arctan \frac{ak}{bh}} =$$

$$= \frac{ab}{6} \left(\frac{a}{h} \right)^2 \left(\frac{1}{1 + \left(\frac{ak}{bh} \right)^2} - 1 \right) + \frac{ab}{6} \left(\frac{b}{k} \right)^2 \left(\frac{1}{1 + \left(\frac{ak}{bh} \right)^2} - 1 \right) =$$

$$= \frac{ab}{6} \left(\frac{a}{h} \right)^2 \frac{(ak)^2 + 3(ak)^2 bh + 3ak(bh)^2}{(ak + bh)^3} + \frac{ab}{6} \left(\frac{b}{k} \right)^2 \frac{-(ak)^2}{(ak + bh)^3} = \frac{c^4 bk \tan^2 \varphi}{6k^2 (ak + bh)^3}$$

11.3.3.1

$$\iint_R X(x) Y(y) dx dy = \int_a^A X(x) dx \cdot \int_b^B Y(y) dy$$

$$\int_a^A dx \int_b^B X(x) Y(y) dy = \int_b^B Y(y) dy \int_a^A X(x) dx$$

12.3.3.1

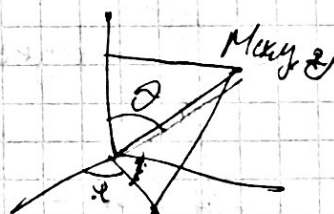
$$\iiint_N f(x, y, z) dx dy dz = \int_a^A dx \int_b^B dy \int_c^C f(x, y, z) dz$$

$$\begin{cases} x = \rho \cos \varphi \\ y = \rho \sin \varphi \\ z = \rho \cos \theta \end{cases}$$

$$\frac{\rho(x, y, z)}{\rho(\varphi, \theta)} = \rho^2 \sin \theta; \quad z = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

$$I = \int_V f(z) dz = 4\pi \int_0^R z^2 f(z) dz$$

$$\begin{cases} x = x(u, v, w) \\ y = y(u, v, w) \\ z = z(u, v, w) \end{cases} \in C^1 \text{ b.d.} \quad \det J \neq 0$$



Задача

$$I = \int_0^x \int_0^y \int_0^z f(x, y, z) dz dy dx = \left| dy = -d(x-y) \right| = - \int_0^x d(x-y) \int_0^y \int_0^z f(x, y, z) dz dy$$

$$I = \int_0^x (x-y) \int_0^y f(x, y, z) dz dy = \int_0^x (x-y) \int_0^y f(x, y, z) dz dy$$

$$d(x-y) = -dy$$

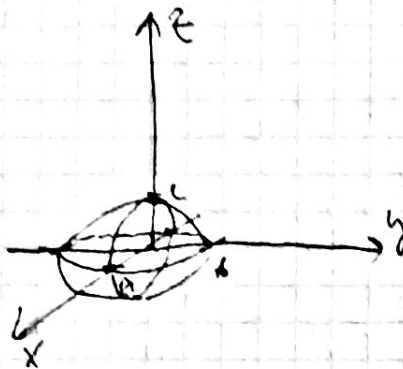
$$\Rightarrow \int_0^x \int_0^y -\frac{1}{2} d(x-y)^2 \int_0^z f(x, y, z) dz dy = -\frac{1}{2} (x-y)^2 \int_0^z f(x, y, z) dz dy =$$

$$= \int_0^x \int_0^y \frac{1}{2} (x-y)^2 f(x, y, z) dz dy$$

Задача

$$I = \iiint_V \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}} dx dy dz$$

$$V: \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$$



Решение. Сделаем замену: $x = ar \cos \varphi$, $y = br \sin \varphi$, $z = cr \cos \theta$

$$x = ar \cos \varphi, \quad y = br \sin \varphi, \quad z = cr \cos \theta$$

$$\left| \frac{\partial(x, y, z)}{\partial(r, \varphi, \theta)} \right| = abc r^2 \sin \theta$$

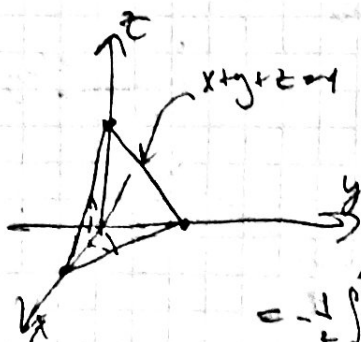
$$I = \int_0^{2\pi} d\varphi \int_0^\pi \sin \theta d\theta \int_0^1 abc r^2 \sqrt{1-r^2} dr = 4\pi abc \int_0^1 r^2 \sqrt{1-r^2} dr$$

$$\int_0^1 r^2 \sqrt{1-r^2} dr = \left| \begin{array}{l} r^2 = \sin^2 \alpha \\ dr = \cos \alpha d\alpha \\ \sqrt{1-r^2} = \cos \alpha \end{array} \right| = \int_0^{\frac{\pi}{2}} \sin^2 \alpha \cos^2 \alpha d\alpha = \frac{1}{4} \int_0^{\frac{\pi}{2}} \sin^2 2\alpha d\alpha = \frac{1}{4} \int_0^{\frac{\pi}{2}} \frac{1 - \cos 4\alpha}{2} d\alpha =$$

$$= \frac{\pi}{16} \Rightarrow I = 4\pi abc \cdot \frac{\pi}{16} = \frac{abc\pi^2}{4}$$

Задача

$$I = \iiint_V \frac{dx dy dz}{(1+x+y+z)^3} = \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} \frac{1}{(1+x+y+z)^3} dz = \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} \frac{d(1+x+y+z)}{(1+x+y+z)^3} \quad (*)$$



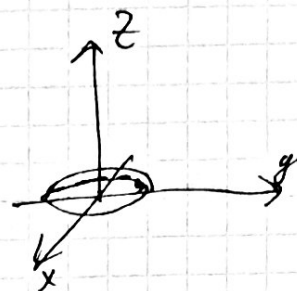
$$= \int_0^1 dx \int_0^{1-x} dy \left[-\frac{1}{2(1+x+y+z)^2} \right]_{z=0}^{z=1-x-y} =$$

$$= -\frac{1}{2} \int_0^1 dx \left(\frac{1}{(1+x)^2} - \frac{1}{(1+x+y)^2} \right) dy = -\frac{1}{8} (1-x) + \frac{1}{2} \int_0^{1-x} \frac{1}{(1+x+y)^2} d(1+x+y) = -\frac{1}{8} (1-x) +$$

$$+ \frac{1}{2} \left(\frac{1}{1+x} - \frac{1}{1+x+y} \right) \Big|_{y=0}^{y=1-x} = \frac{1}{8} (1-x) + \frac{1}{2} \left(\frac{1}{1+x} - \frac{1}{1+x+y} \right) \Big|_{y=0}^{y=1-x} \quad (**)$$

$$\textcircled{2} \int_0^1 \left(\frac{1}{6}(x-1) - \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{1+x} \right) dx = \frac{1}{6}(x-1)^2 - \frac{1}{4}x + \frac{1}{2} \ln|1+x| \Big|_{x=0}^{x=1} = -\frac{1}{16} - \frac{1}{2} \ln 2$$

$$I = \iiint_V z \, dx \, dy \, dz \quad V: \begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \\ z \geq 0 \end{cases}$$



$$V: -\frac{a}{\sqrt{a^2-x^2}} \leq y \leq \frac{b}{\sqrt{a^2-x^2}} \quad 0 \leq z \leq c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}$$

$$\textcircled{2} \int_{-\frac{a}{\sqrt{a^2-x^2}}}^{\frac{b}{\sqrt{a^2-x^2}}} \frac{dz}{2} \Big|_{z=0}^{z=c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}} = \int_{-a}^a dx \int_{-\frac{b}{\sqrt{a^2-x^2}}}^{\frac{b}{\sqrt{a^2-x^2}}} \frac{c^2}{2} \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) dy = \int_{-a}^a \frac{c^2}{2} \left(1 - \frac{x^2}{a^2}\right) y - \frac{y^3}{6} \Big|_{y=-\frac{b}{\sqrt{a^2-x^2}}}^{y=\frac{b}{\sqrt{a^2-x^2}}} dx \quad \textcircled{2}$$

$$\textcircled{2} \int_{-a}^a \left(\frac{c^2}{2} \left(1 - \frac{x^2}{a^2}\right) \left(\frac{b}{\sqrt{a^2-x^2}} \right) - \frac{2 \left(\frac{b}{\sqrt{a^2-x^2}} \right)^3}{6} \right) dx = \int_{-a}^a \frac{c^2 (x^2 - a^2)^{3/2}}{3a^2} \cdot \frac{b}{a} - \frac{2 \frac{b}{a} (x^2 - a^2)^{3/2}}{3a^2} dx =$$

$$= \frac{4bc^2}{3a^3} \int_{-a}^a (a^2 - x^2)^{3/2} dx = \frac{4bc^2}{3a^3} \int_0^a (a^2 - x^2)^{3/2} dx = \frac{4bc^2}{3a^3} \int_0^{\pi/2} (a^4 \cos^4 t) a \cos t dt = \frac{4bc^2 a^2}{3a^3} \int_0^{\pi/2} (1 + \cos 2t)^2 dt$$

$$\textcircled{2} \frac{4bc^2 a^2}{3} \int_0^{\pi/2} \left(\frac{1 + \cos 2t + \cos^2 2t}{2} \right) dt = \frac{4bc^2 a^2}{3} \left(\frac{1}{2} \cdot \frac{\pi}{2} + \int_0^{\pi/2} \left(\frac{1 + \cos 4t}{8} \right) dt \right) = \frac{4bc^2 a^2}{3} \cdot \frac{3\pi}{16} = \frac{bc^2 a^2 \pi}{4}$$

2/p

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$$\iiint_V \frac{dx \, dy \, dz}{(1+x+y+z)^3} \quad V: x+y+z=1, x \geq 0, y \geq 0, z \geq 0$$

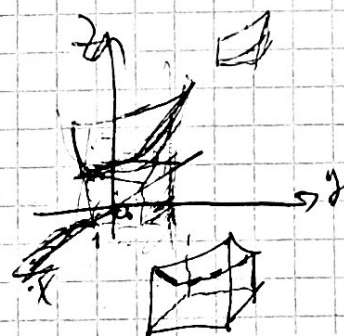


$$\textcircled{2} \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} f(x,y,z) dz \quad \textcircled{2}$$

$$\begin{aligned} 0 &\leq x \leq 1 \\ 0 &\leq y \leq 1-x \\ 0 &\leq z \leq 1-x-y \end{aligned}$$

$$\int_{x^2}^{x^2+1} dz \int_{x^2}^1 f(x,y,z) dy$$

$$+ \int_0^{x^2} dz \int_0^1 f(x,y,z) dy$$



$$0 \leq z \leq x+y$$

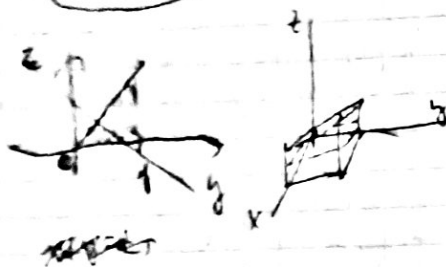
$$\int_0^1 dx \int_x^{x^2} dz \int_{\sqrt{z-x^2}}^1 f(x,y,z) dy$$

$$\textcircled{2} \int_0^1 \left(\int_0^{1-x} \left(\int_0^{1-x-y} f(x,y,z) dz \right) dy + \int_0^1 \left(\int_0^{1-x} f(x,y,z) dy \right) dz + \int_0^1 \left(\int_0^1 f(x,y,z) dz \right) dy \right) dx$$

Ex 15

$$\int_0^1 dx \int_0^{x+1} dy \int_0^{x+y} f(z) dz = \int_0^1 dx \int_0^1 dy \int_0^{x+y} f(z) dz = (-x-1) \int_0^1 dy \int_0^{x+y} f(z) dz \Big|_{x=0}^{x=1} + \int_0^1 (x-1) dx \int_0^{x+1} f(z) dz \quad \text{③}$$

$$\textcircled{=} \int_0^1 dy \int_0^1 f(z) dz + \int_0^1 (x-1) dx \int_0^{x+1} f(z) dz$$



$$I = \int_0^2 dz \iint_{S(z)} f(z) dx dy = \int_0^1 f(z) \frac{1}{2} (2-z)^2 dz + \int_1^2 f(z) \frac{1}{2} (1+z)^2 dz$$

$$S(z) = \begin{cases} z \leq 1: \begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq 1 \\ x+y \geq z \end{cases} \\ z > 1: \begin{cases} x+y \geq z \\ x \leq 1 \\ y \leq 1 \end{cases} \end{cases}$$

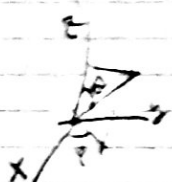
Ex 16

$$I = \iiint_V \sqrt{x^2+y^2+z^2} dx dy dz$$

$$V: \text{cyl. } x^2+y^2+z^2=z, \text{ i.e. } x^2+y^2+z^2-z \leq 0$$

$$\begin{aligned} x &= \rho \sin \theta \cos \varphi \\ y &= \rho \sin \theta \sin \varphi \\ z &= \rho \cos \theta \end{aligned}$$

$$\rho^2 - \rho \cos \theta \leq 0, \quad \rho \leq \cos \theta \Rightarrow \theta \in [0, \frac{\pi}{2}]$$



$$dV = \rho^2 \sin \theta d\rho d\theta d\varphi$$

$$I = \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \sin \theta d\theta \int_0^{\cos \theta} \rho^3 d\rho = \frac{2\pi}{4} \int_0^{\frac{\pi}{2}} \sin \theta \cos^4 \theta d\theta$$

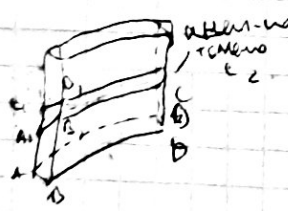
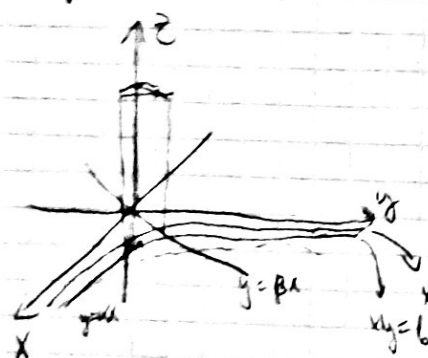
$$\left(\frac{\rho^4}{4} \right) \Big|_0^{\cos \theta} = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \cos^4 \theta d\theta$$

$$\textcircled{=} -\frac{\pi}{2} \left(\frac{\cos^3 \theta}{3} \right) \Big|_0^{\frac{\pi}{2}} = \frac{\pi}{6}$$

Ex 17

$$\iiint_V x y z dx dy dz$$

$$V: x > 0, y > 0, z > 0 \text{ and } \sqrt{z} = \frac{x^2+y^2}{n}, \quad z = \frac{x^2+y^2}{n}, \quad xy = a^2, \quad xy = b^2$$



$$x^2 = a^2, \quad x = \frac{a}{\sqrt{y}}, \quad y = b^2$$

$$A \left(\frac{a}{b}, \sqrt{a} \right), \quad B \left(\frac{b}{a}, \sqrt{b} \right)$$

$$C \left(\frac{a}{b}, \sqrt{a} \right), \quad D \left(\frac{b}{a}, \sqrt{b} \right)$$

$$\frac{z}{x^2+y^2} = u, \quad xy = v, \quad \frac{y}{x} = w, \quad x = \sqrt{\frac{v}{w}}, \quad y = \sqrt{vw}, \quad z = uv(w + \frac{1}{w})$$

$$I = \begin{vmatrix} 0 & \frac{1}{2\sqrt{vw}} & -\frac{\sqrt{v}}{2w\sqrt{w}} \\ \frac{1}{2\sqrt{vw}} & \frac{\sqrt{v}}{2\sqrt{w}} & \frac{\sqrt{v}}{2\sqrt{w}} \\ \frac{v(w+\frac{1}{w})}{2\sqrt{v}} & \frac{u(w+\frac{1}{w})}{2\sqrt{v}} & u\sqrt{1-\frac{1}{w^2}} \end{vmatrix} = \frac{v}{2w(w+\frac{1}{w})}, \quad V: \frac{1}{n} \leq u \leq \frac{1}{m}, \quad a^2 \leq v \leq b^2, \quad \alpha \leq w \leq \beta$$

$$\iiint_V xyz \, dx \, dy \, dz = \int_{\frac{1}{n}}^{\frac{1}{m}} \frac{u}{2} du \int_{a^2}^{b^2} v^{\frac{3}{2}} dv \int_{\alpha}^{\beta} (w + \frac{1}{w} + \frac{z}{w}) dw = \frac{1}{32} \left(\frac{1}{m^2} - \frac{1}{n^2} \right) (b^8 - a^8) \left(\beta^2 - \alpha^2 \right)$$

$$\cdot \left(1 + \frac{1}{\alpha^2 \beta^2} \right) + 4 \ln \frac{\beta}{\alpha}$$

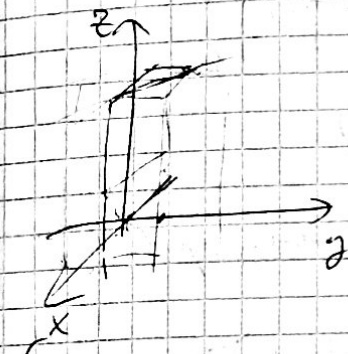
4.10.8

$$z = x+y, \quad z = xy, \quad x+y=1, \quad x=0, y=0, \quad y=1-x, \quad y = \frac{z}{x}, \quad z = x(1-x)$$

$$0 \leq x \leq 1, \quad 0 \leq y \leq 1-x, \quad xy \leq z \leq x(1-x)$$

$$V = \int_0^1 dx \int_0^{1-x} dy \int_{xy}^{x(1-x)} dz =$$

$$= \int_0^1 dx \int_0^{1-x} (x(1-x) - xy) dy = \int_0^1 \left(x(1-x) + \frac{(1-x)^3}{2} \right) dx = \frac{7}{24}$$



4.11.2

$$a \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \right)^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$x = a \rho \sin \alpha \cos \varphi, \quad y = b \rho \sin \alpha \sin \varphi, \quad z = c \rho \cos \alpha$$

$$0 \leq \rho^4 \leq \rho^2 a^2 \sin^2 \alpha$$

$$0 \leq \rho^2 \leq a^2 \sin^2 \alpha$$

$$\rho \leq a \sin \alpha, \quad \alpha \in [0, \frac{\pi}{2}]$$

$$\rho \leq -a \sin \alpha, \quad \alpha \in [\frac{\pi}{2}, \pi]$$

$$\Rightarrow V = \int_0^{2\pi} d\varphi \left(\int_0^{\frac{\pi}{2}} \sin \alpha d\alpha \int_0^{a \sin \alpha} \rho^2 d\rho + \int_{\frac{\pi}{2}}^{\pi} \sin \alpha d\alpha \int_0^{-a \sin \alpha} \rho^2 d\rho \right) + 8 \cdot \int_0^{\frac{\pi}{2}} d\alpha \int_0^{\frac{\pi}{2}} d\varphi \int_0^{a \sin \alpha} \rho^2 \sin \alpha d\rho = \frac{4\pi a^3}{3} \left(\frac{1}{3} \right) \int_0^{\frac{\pi}{2}} \sin^3 \alpha d\alpha = \frac{4\pi a^3}{3} \left(\frac{1}{3} \right) \left(\frac{4}{3} \right) = \frac{16\pi a^3}{27}$$

$$= 8 \cdot \frac{\pi}{2} \frac{abc}{3} \int_0^{\frac{\pi}{2}} \sin^3 \alpha d\alpha = \frac{\pi^2}{4} abc$$

$$\iiint \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \right)^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$$

$$\rho^4 \leq \rho^2 (\sin^2 \alpha - \cos^2 \alpha) = \rho^2 (-\cos 2\alpha), \quad \alpha \in [\frac{\pi}{2}, \pi]$$

$$\Rightarrow \rho \leq \sqrt{-\cos 2\alpha}; \quad V = 8 \cdot \int_0^{\frac{\pi}{2}} d\varphi \int_{\frac{\pi}{2}}^{\pi} d\alpha \int_0^{\sqrt{-\cos 2\alpha}} \rho^2 \sin \alpha d\rho = \frac{abc \cdot 8\pi}{2} \int_{\frac{\pi}{2}}^{\pi} (-\cos 2\alpha)^{\frac{3}{2}} d\alpha = \frac{4\pi abc}{3} \int_{\frac{\pi}{2}}^{\pi} (-\cos 2\alpha)^{\frac{3}{2}} d\alpha$$

$$\Rightarrow \frac{4\pi abc}{3} \int_{\frac{\pi}{2}}^{\pi} \sin \theta (\cos \theta)^{\frac{3}{2}} d\theta = \frac{4\pi abc}{3} \int_0^{\frac{\pi}{2}} \sin \theta (\cos \theta)^{\frac{3}{2}} d\theta = \frac{4\pi abc}{3} \left(\frac{2}{5} \right) = \frac{8\pi abc}{15}$$

$$\textcircled{=} \frac{4abc}{3} \int_0^{\frac{\pi}{2}} \sin^2(\alpha x) dx = \frac{2abc}{3} \int_0^{\frac{\pi}{2}} (1 - \cos 2\alpha x) dx = \frac{\pi^2 abc}{4\sqrt{2}}$$

$\frac{1}{2} \int_0^{\frac{\pi}{2}} \cos 2\alpha x dx$
 $\frac{1}{2} \sin 2\alpha x \big|_0^{\frac{\pi}{2}}$

Краткие упражнения

Задача 1

$$\int_0^1 \int_0^1 \dots \int_0^1 (x_1^2 + x_2^2 + \dots + x_n^2) dx_1 \dots dx_n = \int_0^1 dx_1 \int_0^1 dx_2 \dots \int_0^1 dx_n (x_1^2 + x_n^2) =$$

$$= \int_0^1 dx_1 \int_0^1 dx_2 \dots \int_0^1 dx_{n-1} \left(x_1^2 + \frac{1}{3} \right) = \left(\frac{1}{3} \right) \cdot n$$

Задача 2

$$\int_0^1 \int_0^1 \dots \int_0^1 (x_1 + x_2 + \dots + x_n)^2 dx_1 \dots dx_n = \int_0^1 dx_1 \dots \int_0^1 dx_{n-1} \left(\frac{(x_1 + \dots + x_{n-1})^2}{2} \right) \bigg|_{x_n=0}^{x_n=1}$$

$$\sum_{i,j} x_i^2 + 2 \sum_{i < j} x_i x_j = \frac{n}{3} + 2 \left(\frac{n-1}{4} + \frac{n-2}{4} + \dots + \frac{1}{4} \right) = \frac{n(n+1)}{12}$$

Нормальное распределение n-м. спек. в евр. пр. те (E_n)

$$W_n = \frac{2\pi^{n/2}}{\Gamma(n/2)}$$

$$\Gamma(s) = \int_0^\infty e^{-x} x^{s-1} dx$$

$$\Gamma(s+1) = s \cdot \Gamma(s) = s(s-1)\Gamma(s-1) = \dots = s! \Gamma(1)$$

$$W_n = \frac{2\pi^{n/2}}{\Gamma(n/2)}$$

$$\Gamma'(s) = \int_0^\infty e^{-x} \ln(x) \cdot x^{s-1} dx$$

$$\Gamma'(n+1) = n! ; \Gamma'(\frac{1}{2}) = \sqrt{\pi}$$

$$\Gamma^{(k)}(s) = \int_0^\infty e^{-x} \ln^k(x) x^{s-1} dx$$

$$\Gamma'(\frac{3}{2}) = \Gamma'(1 + \frac{1}{2}) = \frac{1}{2} \Gamma'(\frac{1}{2}) = \frac{\sqrt{\pi}}{2}$$

$$W_1 = \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$\Gamma(\frac{3}{2}) = \Gamma(1 + \frac{1}{2}) = \frac{1}{2} \Gamma(\frac{1}{2}) = \frac{\sqrt{\pi}}{2}$$

$$W_2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2} dx dy = \pi$$

$$\Gamma(n + \frac{1}{2}) = \frac{(2n-1)!!}{2^n} \sqrt{\pi}$$

$$W_n = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} e^{-x_1^2 - \dots - x_n^2} dx_1 \dots dx_n = \pi^{n/2}$$

а) спектр СК:

$$x_1 = \rho \cos \theta_1$$

$$x_2 = \rho \sin \theta_1 \cos \theta_2$$

$$x_n = \rho \sin \theta_1 \dots \sin \theta_{n-1}$$

$$\frac{D(x_1, \dots, x_n)}{D(\rho, \theta_1, \dots, \theta_{n-1})} = \sin^{n-2} \theta_1 \sin^{n-3} \theta_2 \dots \sin \theta_{n-1}$$

$$W_n = \int_0^\infty \rho^{n-1} d\rho \int_0^{2\pi} d\theta_1 \dots \int_0^{2\pi} d\theta_{n-1}$$

$$W_n = \int_0^\infty \rho^{n-1} e^{-\rho^2} d\rho \int_0^{2\pi} d\theta_1 \dots \int_0^{2\pi} d\theta_{n-1} = \left(\int_0^\infty \rho^{n-1} e^{-\rho^2} d\rho \right) \cdot \left(\int_0^{2\pi} d\theta_1 \dots \int_0^{2\pi} d\theta_{n-1} \right) =$$

$$= \frac{1}{2} W_n \int_0^\infty \rho^{n-1} e^{-\rho^2} d\rho, \quad \frac{1}{2} W_n \Gamma(\frac{n}{2}) = \pi^{n/2}, \quad W_n = \frac{2\pi^{n/2}}{\Gamma(n/2)}$$

Оценки n-м. спек.

S4205

$$I_n(a) = \int \int_{x_1 \geq 0, \dots, x_n \geq 0} \int_{x_1 + x_2 + \dots + x_n \leq a} dx_1 dx_2 \dots dx_n = \left| \begin{array}{c} x_1 = a \xi_1 \\ \vdots \\ x_n = a \xi_n \end{array} \right| = a^n \int \int_{\xi_1 \geq 0, \dots, \xi_n \geq 0} \int_{\xi_1 + \dots + \xi_n \leq 1} d\xi_1 d\xi_2 \dots d\xi_n = a^n \cdot \frac{1}{n!} (1)^n$$

$$\textcircled{a} a^n \cdot \int_0^1 d\xi_n \int \dots \int_{\xi_1 \geq 0, \dots, \xi_{n-1} \geq 0} \int_{\xi_1 + \dots + \xi_{n-1} \leq 1 - \xi_n} d\xi_1 \dots d\xi_{n-1} = a^n \cdot 1 \cdot I_{n-1}(1) \quad , \quad I_{n-1}(1) \cdot \int_0^1 (1 - \xi_n)^{n-1} d\xi_n \Rightarrow$$

$$\Rightarrow I_n = \frac{I_{n-1}}{n} = \frac{I_{n-2}}{n(n-1)} = \dots = \frac{I_1}{n!} = 1$$

$$\Rightarrow I_n(a) = \frac{a^n}{n!}$$

S4211 Wozien mapu $R=a$ b E^n

$$x_1^2 + \dots + x_n^2 \leq a^2, \quad V_n = \int \int_{x_1^2 + \dots + x_n^2 \leq R^2} dx_1 \dots dx_n = \left| \begin{array}{c} x_1 = R \xi_1 \\ \vdots \\ x_n = R \xi_n \end{array} \right| = R^n \int \int_{\xi_1^2 + \dots + \xi_n^2 \leq 1} d\xi_1 \dots d\xi_n =$$

$$= R^n \int_0^1 d\xi_n \int \dots \int_{\xi_1^2 + \dots + \xi_{n-1}^2 \leq 1 - \xi_n^2} d\xi_1 \dots d\xi_{n-1} = R^n \int_0^1 (1 - \xi_n^2)^{\frac{n-1}{2}} d\xi_n \cdot \omega_{n-1} \stackrel{(*)}{=} V_n(R) = R^n V_n(1)$$

$$B(p, q) = \int_0^1 x^p (1-x)^q dx = B(q, p) = \frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)} \quad \text{c} \quad \xi_n = z, \quad d\xi_n = \frac{dz}{2iz}$$

$$\textcircled{a} R^n \omega_{n-1} \int_0^1 (1-z^2)^{\frac{n-1}{2}} z^{\frac{1}{2}-1} dz = R^n \omega_{n-1} B\left(\frac{n+1}{2}, \frac{1}{2}\right)$$

$$V_n(R) = R^n \omega_{n-1} \frac{\Gamma(\frac{n+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{n+1}{2} + \frac{1}{2})} = R^n \omega_{n-1} \cdot \frac{1}{n} \quad , \quad V_n(R) = R^n \omega_{n-1} B\left(\frac{n+1}{2}, \frac{1}{2}\right)$$

S4212

$$I_n = \int \int_{x \in \Omega} x_1^2 + \dots + x_n^2 \leq a^2, \quad \Omega: \left\{ \begin{array}{l} x_1^2 + x_2^2 + \dots + x_{n-1}^2 \leq a^2 \\ x_n \geq \frac{a}{2} \end{array} \right.$$

$$I_n = \int_{-\frac{a}{2}}^{\frac{a}{2}} \int \dots \int_{x_1^2 + \dots + x_{n-1}^2 \leq a^2 - x_n^2} dx_1 \dots dx_{n-1} = \int_{-\frac{a}{2}}^{\frac{a}{2}} x_n^2 dx_n (\pi a^2)^{n-1} \omega_{n-2} B\left(\frac{n}{2}, \frac{n}{2}\right) = \left(\frac{a}{2}\right)^{n-1} (\frac{a}{2})^{n-1} \cdot 2 \frac{\pi^{\frac{n-2}{2}}}{\Gamma(\frac{n-2}{2})} B\left(\frac{n}{2}, \frac{n}{2}\right) \cdot \frac{a^3}{12}$$

$$\omega_{n-2} = 2 \frac{\pi^{\frac{n-2}{2}}}{\Gamma(\frac{n-2}{2})}$$

S4213

Klasen sčien n.m. napačljivostnega w eny. granicam:

$$V_n = \int_{-h_1}^{h_1} \int_{-h_2}^{h_2} \dots \int_{-h_n}^{h_n} dx_1 dx_2 \dots dx_n = \left| \begin{array}{c} \xi_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ \vdots \\ \xi_n = a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n \end{array} \right| \quad \text{c} \quad \pm h_i = a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n$$

$$\textcircled{a} \det A \cdot 2^n h_1 \cdot h_2 \dots h_n$$

$$\det A \neq 0$$

Vergleiche Satz: $D_{n+1} \supset D_n$; $I_n = \iint_{D_n} f(x,y) dx dy$, dann folgt I_n
 $\forall D_n \text{ aus } D, D_0$

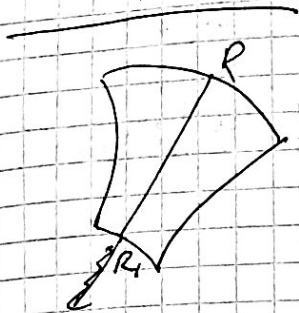
Если $f \neq 0 \Rightarrow$ Крив. Прог. Период
Крив. Дл. 10 минут
интер. послед-но

от нас-е необход.
интервали

Безмисл. а. са. а он са

Если $m \mid n \Rightarrow m \mid \text{ad } m$ (где n - и. инвариант)
 $1 \mid m$ $0 \mid 0$ $0 \mid 0$ $n \neq 1$

Es gilt $\|f(x_1, \dots, x_n)\| \leq \frac{M}{2^2}$, $x_0 = x_1^0, x_2^0, x_4^0$

~~1000~~ - ex.
 $x < n$ 
$$\int_{R_1}^{R_2} r^{n-1} dr, \text{ cell. } n \text{ von } 1 \text{ ab.}$$

254161

$$J = \iint_{x^2+y^2 \geq 1} \varphi(x,y) dx dy, \quad \frac{0}{\sqrt{x^2+y^2}} \leq \varphi(x,y) \leq M, \quad \text{ganz. geb. en. umf.} \therefore \iint_{x^2+y^2 \geq 1} \frac{dx dy}{(x^2+y^2)^p}$$

$$D_n = n^2 \geq x^2 + y^2 \geq 1, \quad y_n = \int_1^{\sqrt{n^2 - x^2}} \frac{dx dy}{(x^2 + y^2)^p} = \left| \begin{array}{l} x = \sqrt{n^2 - y^2} \\ y = \text{const} \\ y = 1 \end{array} \right| = \int_0^{\sqrt{n^2 - 1}} \frac{1}{1 + y^2} dy = \int_1^{\sqrt{n^2 - 1}} \frac{dy}{1 + y^2} = \int_1^{\sqrt{n^2 - 1}} \frac{dy}{y^2 + 1} =$$

$$2 \cdot 2^q \cdot \frac{1}{2-2^q} - \frac{1}{2^{2q}-2} \Big|_1^n = \frac{2n}{2-2^q} \left(\frac{1}{n^{2^q}} - 1 \right)$$

4m. ex $2p-2 > 0, p > 1$. | ~~$2p-2 > 1$~~ 3 ca.

$$Y = \lim_{n \rightarrow \infty} \frac{\pi}{(1-p)} \left(\frac{1}{n^p} - 1 \right) = \frac{\pi}{p-1}$$

24191

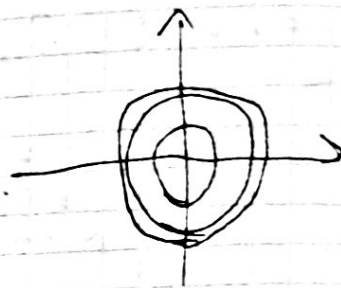
$$I = \int_{x^2+y^2 \leq 1} \frac{e^{ax,y,z}}{(x^2+y^2+z^2)^p} dx dy dz, \quad D_n = \{1 \leq x^2+y^2+z^2 \leq n^2\}, \quad D_n \subset D_{n+1}, \quad \bigcup_n D_n = \{x^2+y^2+z^2 \geq 1\}$$

$$V = \iiint_{1 \leq x^2 + y^2 + z^2 \leq 4} \frac{dx dy dz}{(x^2 + y^2 + z^2)^{3/2}} = \left| \begin{array}{l} x = \rho \cos \varphi \\ y = \rho \sin \varphi \\ z = \rho \cos \alpha \\ 0 \leq \varphi \leq 2\pi \\ 0 \leq \alpha \leq \pi \\ \rho = r \sin \alpha \end{array} \right| = \underbrace{\int_0^{2\pi} d\varphi \int_0^\pi \sin \alpha d\alpha}_{4\pi} \int_1^2 \frac{\rho^2}{\rho^3} d\rho = 4\pi \int_1^2 \frac{1}{\rho} d\rho$$

5462

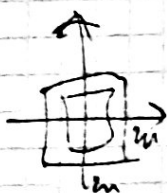
$$I = \iint_{E^2} \sin(x^2+y^2) dx dy, \quad D_n: x^2+y^2 \leq 2n \text{ and } 2n-1$$

$$E^2 \quad I_n = \iint_{x^2+y^2 \leq 2n} \sin(x^2+y^2) dx dy = \begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \\ r = \sqrt{2} \\ 0 \leq \varphi \leq 2\pi \\ 0 \leq r \leq \sqrt{2n} \end{cases}$$



$$\textcircled{=} I_n = \int_0^{2\pi} d\varphi \int_0^{\sqrt{2n}} \sin(r^2) r dr = 2\pi \cdot \frac{1}{2} \int_0^{2n} \sin(t) dt = \pi \cos t \Big|_0^{2n} = \pi(1 - \cos(2n)) = 0$$

$$2) D_n: |x| \leq n, |y| \leq n$$



$$I_n = \iint_{|x| \leq n, |y| \leq n} \sin(x^2+y^2) dx dy = \iint_{|x| \leq n, |y| \leq n} (\sin x^2 \cos y^2 + \sin y^2 \cos x^2) dx dy = 8 \int_0^n \sin x^2 dx \int_0^n \cos y^2 dy$$

$$\textcircled{=} 8 \int_0^n \sin x^2 dx \int_0^n \cos y^2 dy = \int_0^n \sin x^2 dx = \left| \begin{matrix} x^2 = z \\ x = \sqrt{z} \\ dx = \frac{dz}{2\sqrt{z}} \end{matrix} \right| = \frac{1}{2} \int_0^{n^2} \frac{\sin z}{\sqrt{z}} dz$$

$$\textcircled{=} \frac{1}{2} \int_0^{n^2} \frac{\sin z}{\sqrt{z}} dz = \frac{1}{2} \int_0^{n^2} \frac{\sin z}{\sqrt{z}} dz - \text{as } n \rightarrow \infty \text{ Fresnel}$$

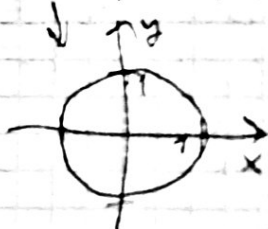
$$\frac{1}{2} \int_0^{n^2} \frac{\cos z}{\sqrt{z}} dz = \frac{1}{2} \int_0^{n^2} \frac{\cos z}{\sqrt{z}} dz \xrightarrow{n \rightarrow \infty} \frac{1}{2} \sqrt{\frac{\pi}{2}}; \textcircled{=} 8 \cdot \frac{1}{4} \sqrt{\frac{\pi}{2}} = \pi$$

Универс. пак.

4.171

$$I = \iint_{x^2+y^2 \leq 1} \frac{dx dy}{\sqrt{1+x^2+y^2}}$$

Универс. пак.



$$D_n = x^2+y^2 \leq \left(1 - \frac{1}{n}\right)^2, \quad D_n \subset D_{n+1}, \quad \bar{D}_n \subset \bar{D}_{n+1} \cup \bar{D}_n = \emptyset$$

() is not
see r. as.

$$I_n = \iint_{x^2+y^2 \leq \left(1 - \frac{1}{n}\right)^2} \frac{dx dy}{\sqrt{1+x^2+y^2}} = \begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \\ r = 1 - \frac{1}{n} \\ 0 \leq \varphi \leq 2\pi \\ 0 \leq r \leq 1 - \frac{1}{n} \end{cases} = \int_0^{2\pi} d\varphi \int_0^{1 - \frac{1}{n}} \frac{r dr}{\sqrt{1+r^2}} = 2\pi \cdot \frac{1}{2} \int_0^{1 - \frac{1}{n}} \frac{dr}{\sqrt{1+r^2}} = \pi \sqrt{1+r^2} \Big|_0^{1 - \frac{1}{n}} = 2\pi \left(1 - \sqrt{1 - \frac{1}{n}}\right)$$

4.174

$$I = \iint_{0 \leq x \leq y} e^{-x-y} dx dy = \int_0^\infty dx \int_x^\infty e^{-x-y} dy = \int_0^\infty \frac{dx}{e} \left(\frac{dx}{e} \right) = \int_0^\infty \frac{dx}{e^2} = \frac{1}{2}$$

4.175

$$I = \iint_{-\infty < x < y < \infty} e^{-x-y} dx dy = \int_{-\infty}^\infty y \cdot \int_{-\infty}^y e^{-x-y} dx dy = \int_{-\infty}^\infty y \cdot \frac{dx}{e} = \int_{-\infty}^\infty y \cdot \frac{dy}{e} = \pi$$

Универс. пак.

$$I = \iiint_{x^2+y+z \leq 1} dx dy dz$$
$$\vec{r}_n = \begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \\ \rho = r \sin \theta \end{cases}$$

$$\begin{aligned} & \alpha \leq \beta \leq \pi \\ & \alpha \leq \varphi \leq \pi, \\ & \alpha \leq \sigma \leq (1 - \frac{1}{n}) \end{aligned}$$

$$\textcircled{=}\ 4\pi \int_0^{\frac{\sqrt{e}}{2}} \frac{\sin^2 x}{(\cos x)^p} dx$$



$$f(r) = k = \int_0^1 \frac{r^2 dr}{(1-r)^p} = \left| \begin{matrix} r^2 = t \\ r = \sqrt{t} \\ dr = \frac{dt}{2\sqrt{t}} \end{matrix} \right| = \int_0^1 \frac{t dt}{2\sqrt{t}(1-t)^p} = \int_0^1 \frac{\sqrt{t} dt}{2(1-t)^p} = \frac{1}{2} B\left(\frac{1}{2}, 1-p\right)$$

$$4\pi \frac{1}{2} \frac{\Gamma(\frac{3}{2})\Gamma(1-p)}{\Gamma(\frac{5}{2}-p)}.$$

4162

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{f(x,y)}{(1+x^2)^p (1+y^2)^q} dx dy \quad (2)$$

$$S_n = \{0 \leq |x| \leq n, |y| \leq n\} \Rightarrow$$

$$T_n = \iint_{\substack{|x| \leq n \\ |y| \leq n}} \frac{dx dy}{(1+|x|^p)(1+|y|^q)} = \int_{|x|=0}^n \left(\int_{|y| \leq n} \frac{dy}{(1+|y|^q)} \right) \frac{dx}{(1+|x|^p)} =$$

$$\textcircled{2} \quad 4 \left(\int_0^{+\infty} \frac{dx}{1+x^2} \right) \left(\int_0^{+\infty} \frac{dy}{1+y^2} \right)$$

$$\int_{0 \leq y \leq n} \frac{dy}{(1+y^2)} = \arctan y^2 \Big|_{y=0}^{y=n}$$

$$\lim_{x \rightarrow \infty} x^p \cdot \frac{1}{1+x^p} = 1, \Rightarrow \begin{matrix} p > 1 & \text{ca.} \\ p \leq 1 & \text{pære.} \end{matrix} \quad \text{c} \quad \text{q} \quad \text{afkastet}$$

$$\iint_{0 \leq y < 1} \frac{p(x,y)}{(1+x^2+y^2)^p} dx dy \approx 2 \cdot \int_0^1 dy \int_0^{+\infty} \frac{dx}{(1+x^2+y^2)^p}, \quad 0 \leq y < 1: \int_0^{+\infty} \frac{dx}{(2+x^2)^p} \approx \int_0^{+\infty} \frac{dx}{(1+x^2+y^2)^p} \leq \int_0^{+\infty} \frac{dx}{(1+x^2)^p} \quad (p \geq 0).$$

$$\lim_{x \rightarrow \infty} x^p \cdot \frac{1}{(a^2 + x^2)^p} = 1 \Rightarrow \int_0^{+\infty} \frac{1}{(a^2 + x^2)^p} dx \quad \text{ca. } p > \frac{1}{2}, \text{ per } p \leq \frac{1}{2} \Rightarrow$$

4165

$\iint_{x+y>1} \frac{\sin x \sin y}{(x+y)^p} dx dy$, $\mathcal{R}_n = \{(x,y) \mid 1 \leq x+y \leq 2n, -2n \leq x-y \leq 2n\}$ $\mathcal{R}'_n = \{(x,y) \mid 1 \leq x+y \leq 2n, -2n \leq x-y \leq 2n\}$
 $p < 1$ par.



$\sin x \sin y = \frac{1}{2} (\cos(x-y) - \cos(x+y))$

$x-y=0 \quad \begin{cases} x = \frac{u+v}{2} \\ y = \frac{u-v}{2} \end{cases} ; \frac{D(x,y)}{D(u,v)} = -\frac{1}{2}$

$\mathcal{R}'_n = \{(x,y) \mid 2n - \frac{\pi}{4} \leq x+y \leq 2n, -2n \leq x-y \leq 2n\}$

$\lim_{n \rightarrow \infty} \iint_{\mathcal{R}_n} \frac{\sin x \sin y}{(x+y)^p} dx dy = \lim_{n \rightarrow \infty} \left(\iint_{\mathcal{R}_n} \frac{\sin x \sin y}{(x+y)^p} dx dy - \iint_{\mathcal{R}'_n} \frac{\sin x \sin y}{(x+y)^p} dx dy \right)$

$= 0 ; \quad \frac{1}{4} \int_{2n-\frac{\pi}{4}}^{2n} du \int_{-2n}^{2n} \frac{\cos v - \cos u}{u^p} dv = -n \int_{2n-\frac{\pi}{4}}^{2n} \frac{\cos u}{u^p} du \geq$

$\geq \frac{1}{4} \int_{2n-\frac{\pi}{4}}^{2n} \frac{du}{u^p} \geq \begin{cases} \frac{\pi}{4^{1/2}} \frac{1}{(2n)^p}, & p > 0 \\ \frac{\pi}{4^p}, & p \leq 0 \end{cases}$

$p \leq -1 : \lim_{n \rightarrow \infty} \iint_{\mathcal{R}_n} \frac{\sin x \sin y}{(x+y)^p} dx dy = \infty$

$p > -1 \rightarrow \infty \Rightarrow$ par.

4126

$\iint_{-\infty}^{\infty} e^{-x^2 y^2} \cos(x^2 y^2) dx dy = \left| \begin{matrix} x = r \cos \theta \\ y = r \sin \theta \end{matrix} \right| = \frac{1}{2} \int_0^{2\pi} d\theta \int_0^{\infty} e^{-r^2} \cos r^2 dr^2$

$= \pi \left(\frac{\sin t - t \cos t}{1+t^2} \right) \Big|_0^{\infty} = \frac{\pi}{2}$

$\int_0^{\infty} e^{-x} \cos x dx = \frac{1}{2} \int_0^{\infty} e^{-x} (\cos x + \sin x) dx$
 $= \frac{1}{2} \left(\frac{-\sin x - \cos x}{1+1} \right) \Big|_0^{\infty} = \frac{\pi}{2}$

4177

$\iint_{-\infty}^{\infty} e^{-x^2 y^2} \sin(x^2 y^2) dx dy = \int_0^{2\pi} d\theta \int_0^{\infty} r e^{-r^2} \sin r^2 dr$

$= \pi \left(\frac{-\sin t - t \cos t}{1+t^2} \right) \Big|_0^{\infty} = \frac{\pi}{2}$

4195

$\iiint_{|x|+|y|+|z| \geq 1} \frac{dx dy dz}{x^p + y^q + z^r}$, $p > 0, q > 0, r > 0$

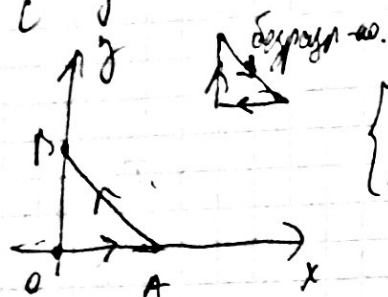
$= 8 \iint_{x>0, y>0, z>0, x+y+z \geq 1} \frac{dx dy dz}{x^p + y^q + z^r} = 8 \left(\iint_{\mathcal{R}_1} + \iint_{\mathcal{R}_2} \right)$

$\mathcal{R}_1 = \{(x,y,z) \mid x>0, y>0, z>0, x+y+z \geq 1, x^p + y^q + z^r \leq 1\}$

$\mathcal{R}_2 = \{ \dots \}$

$\mathcal{R}_3 = \{(x,y,z) \mid x>0, y>0, z>0, x^p + y^q + z^r \geq 1\}$

$$\int_C (x+y) dS, \quad C: \text{конус } \pi \text{ с верш. } A(0,0)$$



$$\begin{cases} x=t, & 0 \leq t \leq 1 \\ y=0 \end{cases}$$

$$\begin{cases} x=t, & 0 \leq t \leq 1 \\ y=t \end{cases}$$

$$\begin{cases} x=0 \\ y=t \end{cases} \quad -1 \leq t \leq 0$$

$$\int_C (x+y) dS = \int_0^1 t dt + \int_0^1 \sqrt{2} t dt + \int_{-1}^0 -t dt = 1 + \sqrt{2}$$

$$\begin{cases} x+y=t, & \text{OA} \\ x+y=1, & \text{AB} \\ x+y=-t, & \text{OB} \end{cases} \quad dS = \sqrt{x'(t)^2 + y'(t)^2} dt$$

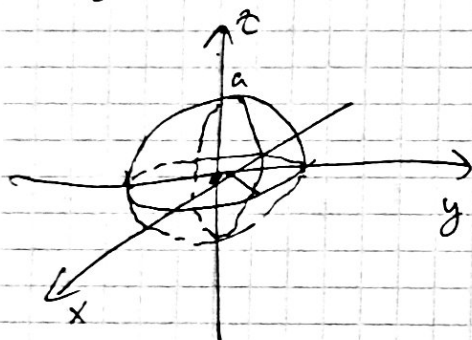
4281

$$\int_C (y-z) dx + (x-z) dy + (x-y) dz$$

$$C: \text{окр. } x^2 + y^2 + z^2 = a^2$$

$$y = x \tan \alpha, \quad 0 \leq \alpha \leq \pi$$

прямая радиус. сечения
то е. нормаль.



$$\begin{aligned} x &= a \cos \alpha \cos \varphi \\ y &= a \sin \alpha \cos \varphi \\ z &= a \sin \alpha \end{aligned}$$

$$dx = -a \sin \alpha \sin \varphi d\varphi$$

$$dy = -a \sin \alpha \cos \varphi d\varphi$$

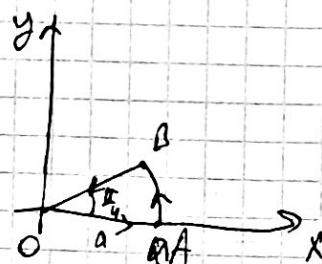
$$dz = a \cos \alpha d\alpha$$

4226

$$I = \int_C e^{a\sqrt{x^2+y^2}} dS$$

$$C: \begin{cases} \varphi=0 \\ \varphi=\pi/4 \end{cases}$$

$$dS = \sqrt{(x'(t))^2 + (y'(t))^2} dt = \sqrt{r^2(\varphi) + (r'(\varphi))^2} d\varphi$$



$$OA: \begin{cases} z=t \\ \varphi=0, & 0 \leq t \leq a \end{cases}$$

AB:

$$I = \int_0^a e^{ar} dr + \int_0^{\pi/4} e^{a r} a dr + \int_0^a e^{a r} dr = 2e^a - 1 + \frac{\pi}{4} a e^a$$

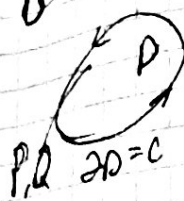
4258

$$\int_{(1,2)}^{(2,1)} x dy + y dx = \int dx y =$$

4257

$$\int_{(0,1)}^{(1,1)} x dx + y dy$$

$$\oint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_D P dx + Q dy$$



Безв. поле - век. , если fulfillment: $du = Pdx + Qdy$, тогда

Крит. Безв. поле не является:

$$a) \int P dx + Q dy = 0$$

$$\text{неод. } \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

Пример

$$\sqrt{4260} \int_{(0,1)}^{(2,3)} (x+y) dx = (x-y) dy = \int_{(0,1)}^{(2,3)} x dx + y dx - x dy + y dy = \int_{(0,1)}^{(2,3)} d\left(\frac{x^2}{2} + \frac{y^2}{2} + y^2 d\left(\frac{x}{y}\right)\right) = \dots$$

+

2/1

4222

$\int_C y^2 ds$, C - arc of a circle

$$x = a \cos t, y = a \sin t \quad (0 \leq t \leq 2\pi)$$

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{a^2 \sin^2 t + a^2 \cos^2 t} dt = a dt$$

$$\int_C y^2 ds = \int_0^{2\pi} a^3 \sin^2 t dt = 8a^3 \int_0^{2\pi} \sin^2 \frac{t}{2} dt = 32a^3 \int_0^{\pi} \sin^2 u du = \frac{256}{15} a^3$$

4225

$$\int_C (x^{\frac{1}{3}} + y^{\frac{1}{3}}) ds, ds = \sqrt{1+y'^2} dx = \frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}} dx$$

$$4 \int_0^a (x^{\frac{1}{3}} + (a^{\frac{1}{3}} - x^{\frac{1}{3}})^2) \frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}} dx = 4a^{\frac{1}{3}} \int_0^a (2x + a^{\frac{1}{3}} x^{-\frac{1}{3}} - 2a^{\frac{2}{3}} x^{\frac{1}{3}}) dx = 4a^{\frac{1}{3}}$$

4222

$$\int_C |y| ds, C: (x^2 + y^2)^2 = a^2 (x^2 - y^2), r^2 = a^2 \cos 2\varphi, ds = \sqrt{r'^2 + r^2} d\varphi = \frac{a}{\sqrt{\cos 2\varphi}} d\varphi$$

$$4 \int_0^{\frac{\pi}{4}} a \sqrt{\cos 2\varphi} \sin \varphi \cdot \frac{a}{\sqrt{\cos 2\varphi}} d\varphi = 4a^2 (-\cos \varphi) \Big|_0^{\frac{\pi}{4}} = 2a^2 (2 - \sqrt{2})$$

4230

$$\int_C (x^2 - 2xy) dx + (y^2 - 4xy) dy, C: y = x^2 \quad (-1 < x < 1); dy = 2x dx$$

$$\int_{-1}^1 ((x^2 - 2x^3) + 2x(x^4 - 4x^3)) dx = -\frac{14}{15}$$

Задача

1) $y = 1 - (1-x) = x$; $0 \leq x \leq 1$; $dy = dx$

$$\int_0^1 (x^2 + (1-x)^2 - x^2 + (1-x)^2) dx = \frac{2}{3} \quad 1 \leq x \leq 2 : y = 1 - (x-1) = 2-x, \quad dy = -dx$$

Задача

$$\int_{\gamma} (x-y)(dx-dy) , (x-y)(dx-dy) = d\left(\frac{(x-y)^2}{2}\right) ; \Rightarrow I = -2.$$

Задача

$$\int_{\gamma} f(x,y) dx + g(x,y) dy ; F(x,y) = \int_0^x f(u,y) du , F'_x(x,y) = f(x,y) ,$$

$$F'_y(x,y) = g(x,y) ; dF(x,y) = F'_x(x,y) dx + F'_y(x,y) dy = f(x,y) dx + g(x,y) dy$$

$$\int_{\gamma} f(x,y) dx + g(x,y) dy = F(a,b) - F(a,0) = \int_0^b f(u) du$$

кр. задачи

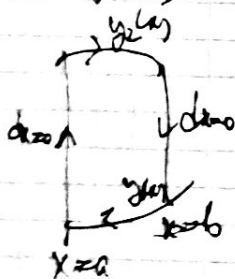
$$(P(x,y), Q(x,y)) \in C^1(D) ; \text{Уп. } \partial D = \Gamma = \gamma = C ; \bar{D} = D \cup \Gamma$$

$$\oint_D \left(P \frac{\partial Q}{\partial x} - Q \frac{\partial P}{\partial y} \right) dx dy = \oint_{\Gamma} P dx + Q dy$$

Убедитесь, что формула верна

Проверка формулы

$$D = \bar{D} , \Gamma = \partial D$$



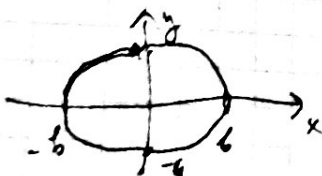
$$\iint_D \frac{\partial P}{\partial y} dx dy = - \int_C P dx , \iint_D \frac{\partial Q}{\partial x} dx dy = \int_C Q dy$$

$$\oint Q = x, P = 0 \Rightarrow \oint_D dx dy = \oint_C x dy ; Q = 0, P = x - y ; \oint_D (y dx - x dy) \Rightarrow |D| = \frac{1}{2} \int_C x dy - y dx$$

Задача

$$x = a \cos t , y = b \sin t , 0 \leq t \leq 2\pi$$

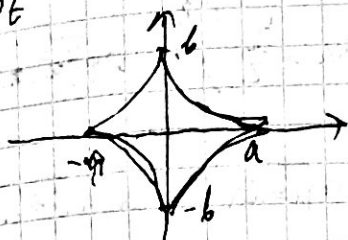
$$|D| = \frac{1}{2} \int_C x dy - y dx = \frac{1}{2} \int_0^{2\pi} (a \cos t) b \cos t + b \sin t a \sin t dt = \frac{1}{2} \int_0^{2\pi} ab dt = \pi ab$$



$$\iint_{\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1} dx dy = 4 \cdot \int_0^a dx \int_0^{\frac{b}{a} \sqrt{1 - \frac{x^2}{a^2}}} dy = 4b \int_0^a \sqrt{1 - \frac{x^2}{a^2}} dx = 4b \int_0^{\frac{\pi}{2}} a \cos^2 t dt = 4ba \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2t}{2} dt = \pi ab$$

4.10.03

$$x = a \cos t \\ y = b \sin t$$

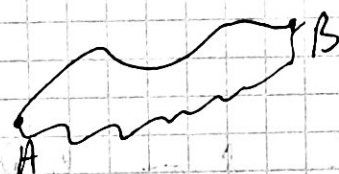


ошибка Лапласа. упр-е адекватно.

$$\frac{1}{4} = \frac{1}{2\pi} \oint_C x dy - y dx = \frac{1}{2\pi} \int_0^{\frac{\pi}{2}} \frac{1}{4} ab \sin^2 t (1 \cos t + \cos t) dt = \frac{1}{4} \Rightarrow \frac{1}{2} \pi ab = \frac{3}{8} \pi ab$$

Проверка нахождение
крив. групп

$$\int_{AB} P dx + Q dy$$



Не забываете на этапе AB?
(А) (сначала от 1.)

Замечание к формуле

$$\oint_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

→ 1

$$① \int_C P dx + Q dy = 0 \text{ и } \text{выбранный, и не}$$

$$\forall C \in D$$

$$② P dx + Q dy = du(x, y)$$

$$③ P dx + Q dy = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

$$P = \frac{\partial u}{\partial x}, Q = \frac{\partial u}{\partial y} \text{ тогда } \Rightarrow \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

$$④ \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

Доказательство: 1 → 2 → 3 → 4 → 1

Эквивалентность
формулы
Грунвальда-Олемского

4.10.10

$$(x+y)^2 = ax, a > 0$$

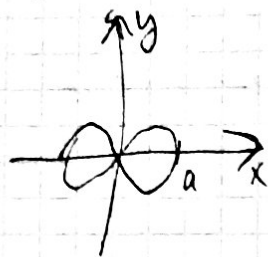
$$\begin{cases} (x+y)^2 = ax \\ y=0 \end{cases} \Rightarrow y = tx, t \in (0, +\infty) \\ x = \frac{a}{(1+t)^2} \Rightarrow y = \frac{at}{(1+t)^2}$$

$$I = \int_C x dy - y dx = \frac{1}{2} \int_0^{+\infty} \frac{a}{(1+t)^2} \cdot \frac{d(at)}{(1+t)^2} - \frac{at}{(1+t)^2} d\left(\frac{a}{(1+t)^2}\right) = \frac{a^2}{2} \int_0^{+\infty} \frac{(1+t)^2 - 2at + at^2}{(1+t)^5} + \frac{2t}{(1+t)^2} \cdot \frac{2a}{(1+t)^3} dt = \frac{a^2}{2} \int_0^{+\infty} \frac{1}{(1+t)^5} (1+t^2+2at) dt = \frac{a^2}{2} \int_0^{+\infty} \frac{1}{(1+t)^5} dt = \frac{a^2}{2} \left[-\frac{1}{4(1+t)^4} \right]_0^{+\infty} = \frac{a^2}{8}$$

$$= \frac{1}{2} a^2 \cdot \frac{1}{(1+t)^3} \left(\frac{1}{-t} \right) \Big|_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{1}{6} a^2.$$

Задача 12

$$(x^2 + y^2)^2 = a^2(x^2 - y^2)$$



$$\begin{aligned} x &= r \cos \varphi \\ y &= r \sin \varphi \\ r^2 &= a^2 \cos 2\varphi \\ r &= a \sqrt{\cos 2\varphi} \end{aligned}$$

$$\begin{aligned} x &= a \sqrt{\cos 2\varphi} \cos \varphi \\ y(\varphi) &= a \sqrt{\cos 2\varphi} \sin \varphi \end{aligned}$$

$$I = \frac{1}{2} \oint_C x dy - y dx = \frac{1}{2} \int_0^{2\pi} a \sqrt{\cos 2\varphi} (\cos^2 \varphi - \sin^2 \varphi) d\varphi$$

$$\begin{aligned} &\rightarrow a \sqrt{\cos 2\varphi} \cos \varphi d(a \sqrt{\cos 2\varphi} \cos \varphi) - a \sqrt{\cos 2\varphi} \sin \varphi d(a \sqrt{\cos 2\varphi} \sin \varphi) \\ &= \frac{1}{2} a^2 \cos 2\varphi \cdot a \left(\frac{1}{2} \sin 2\varphi \cos 2\varphi + \frac{1}{2} \sin^2 2\varphi \right) \end{aligned}$$

Задача 21

C : контур

$\vec{r}$ — радиус-вектор; $|\vec{r}| = 1$; $\vec{r} = (\cos \varphi, \sin \varphi)$



$$(\vec{r}, \vec{n}) = \cos(\vec{r}, \vec{n}), \quad \oint_C (\vec{r}, \vec{n}) = \oint_C \left(\frac{\partial}{\partial x} (\cos \varphi) - \frac{\partial}{\partial y} (\sin \varphi) \right) ds = 0$$

Задача 34

$$\iint_D (u \frac{\partial v}{\partial x} - v \frac{\partial u}{\partial y}) dx dy = \oint_C (u \frac{\partial v}{\partial x} - v \frac{\partial u}{\partial y}) ds, \quad \vec{n} \text{ — нормаль к } C;$$

$u(x, y), v(x, y)$

$u, v \in C^2(D)$

$$\oint_C u \frac{\partial v}{\partial x} ds = \oint_C u \left(\frac{\partial v}{\partial x} \cos(\vec{n}, x) + \frac{\partial v}{\partial y} \cos(\vec{n}, y) \right) ds$$

$$\begin{aligned} dy &= \cos(\vec{n}, y) ds \\ dx &= \cos(\vec{n}, x) ds \end{aligned} \quad \Leftrightarrow \oint_C u \left(\frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy \right) \Leftrightarrow$$

$$P = u \frac{\partial v}{\partial x} = \frac{\partial}{\partial x} (u v) - v \frac{\partial u}{\partial x}$$

$$\oint_C \left(\frac{\partial}{\partial x} (u v) - v \frac{\partial u}{\partial x} \right) dx$$

$$Q = v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} (u v) - u \frac{\partial v}{\partial y}$$

=

8/13

2/3



$x^2 + y^2 = R^2$, $R^2 = x^2 + y^2$, $x = R \cos \varphi$; $y = R \sin \varphi$; $r^2(\cos^2 \varphi + \sin^2 \varphi) = r^2$, $r = \cos^2 \varphi + \sin^2 \varphi$ (2)
 $x^2(x-1) + y^2(y-1) = 0$
 $\Rightarrow \oint_C x dy - y dx = \frac{1}{2} \oint_C (x dy - y dx) = \frac{1}{2} \int_0^{2\pi} R^2 d\varphi$

$\odot \begin{cases} x = (\cos^2 \varphi + \sin^2 \varphi) \cos \varphi \\ y = (\cos^2 \varphi + \sin^2 \varphi) \sin \varphi \end{cases}$; $dx = (-2 \sin \varphi (\cos^2 \varphi + \sin^2 \varphi) + 2 \cos \varphi (-\cos^2 \varphi \sin \varphi + \sin^2 \varphi \cos \varphi))$
 $dy = \cos \varphi (\cos^2 \varphi + \sin^2 \varphi) + 2 \sin \varphi (-\cos^2 \varphi \sin \varphi + \sin^2 \varphi \cos \varphi)$

$\Rightarrow x dy - y dx = (\cos^2 \varphi + \sin^2 \varphi) \cos \varphi (2 \cos^2 \varphi \sin \varphi + 2 \sin^2 \varphi \cos \varphi) - (\cos^2 \varphi + \sin^2 \varphi) \sin \varphi (2 \sin^2 \varphi \cos \varphi - 2 \cos^2 \varphi \sin \varphi) =$
 $= (\cos^2 \varphi + \sin^2 \varphi) (2 \cos^3 \varphi \sin \varphi + 2 \sin^3 \varphi \cos \varphi - 2 \sin^3 \varphi \cos \varphi + 2 \cos^3 \varphi \sin \varphi) =$
 $= (\cos^2 \varphi + \sin^2 \varphi) (4 \cos^3 \varphi \sin \varphi) = 4 \cos^3 \varphi \sin \varphi$
 $\int_0^{2\pi} 4 \cos^3 \varphi \sin \varphi d\varphi = 4 \int_0^{2\pi} \cos^3 \varphi \sin \varphi d\varphi = 4 \left[-\frac{\cos^4 \varphi}{4} \right]_0^{2\pi} = 0$

$y = 2bx$, $x = \frac{1+t^2}{1+t^3}$, $y = \frac{t(1+t^2)}{1+t^3}$ ($0 \leq t < +\infty$)

$x dy - y dx = \frac{(1+t^2)^2}{(1+t^3)^2} dt$, ($0 \leq t < +\infty$)

$S = \frac{1}{2} \int_0^{+\infty} \frac{(1+t^2)^2}{(1+t^3)^2} dt = \frac{1}{2} \left(\int_0^{+\infty} \frac{t^4}{(1+t^3)^2} dt + 2 \int_0^{+\infty} \frac{t^2}{(1+t^3)^2} dt + \int_0^{+\infty} \frac{1}{(1+t^3)^2} dt \right) =$

$= \frac{1}{2} \left(\frac{1}{3} B(2, \frac{1}{3}) + \frac{2}{3} B(1, 1) + \frac{1}{3} B(\frac{1}{3}, 2) \right) = \frac{1}{3} \frac{2}{3} \frac{\pi}{\sin \frac{\pi}{3}} = \frac{1}{3} + \frac{4\pi}{9\sqrt{3}}$

$B(p, q) = \int_0^1 x^{p-1} (1-x)^{q-1} dx = \frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)}$
 $p > 0, q > 0$

8/14

$(x+y)^{n+1} = ax^ny^n$ ($a > 0, n > 0, m > 0$)

$y = tx \Rightarrow \begin{cases} x = \frac{at^m}{(1+t)^{n+m+1}} \\ y = \frac{at^{m+1}}{(1+t)^{n+m+1}} \end{cases}$ ($0 \leq t < +\infty$), $x dy - y dx = \frac{a^2 t^{2m}}{(1+t)^{2n+2m+2}} dt$ ($0 \leq t < +\infty$)

$S = \frac{1}{2} \oint_C x dy - y dx = \frac{a^2}{2} \int_0^{+\infty} \frac{t^{2m}}{(1+t)^{2n+2m+2}} dt = \frac{a^2}{2} B(2m+1, 2n+1)$

8/15

$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1$ ($a > 0, b > 0, n > 0$) - ovals

$x = a \cos^{\frac{2}{n}} \varphi$, $y = b \sin^{\frac{2}{n}} \varphi$ $0 \leq \varphi \leq \frac{\pi}{2}$

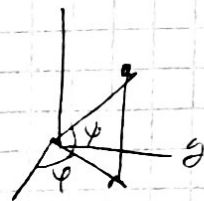
$$x dy - y dx = \frac{2ab}{n} \cos^{\frac{n}{2}-1} \varphi \sin^{\frac{n}{2}-1} \varphi d\varphi$$

$$S = \frac{1}{2} \oint_C x dy - y dx = \frac{1}{2} \int_0^{2\pi} \frac{2ab}{n} \cos^{\frac{n}{2}-1} \varphi \sin^{\frac{n}{2}-1} \varphi d\varphi = \frac{ab}{n} \cdot \frac{1}{2} B\left(\frac{1}{n}, \frac{1}{n}\right) = \frac{ab}{2n} \frac{\Gamma(\frac{1}{n}) \Gamma(\frac{1}{n})}{\Gamma(\frac{2}{n})}$$

Σ 4216

1) $\left(\frac{x}{a} + \frac{y}{b} + \frac{z}{c}\right)^2 = \frac{x}{a} + \frac{y}{b} \quad (x > 0, y > 0, z > 0)$

$$\begin{aligned} x &= ar \cos^2 \theta \cos \varphi \\ y &= br \cos^2 \theta \sin \varphi \\ z &= cr \sin^2 \theta \end{aligned}$$



$$\frac{1}{c} + \cos^2 \theta \left(\frac{a}{h} - \frac{b}{c} \right)$$

$$\frac{\partial \rho}{\partial x} = 4abc r^2 \cos \varphi \sin \varphi \cos^3 \theta \sin \theta$$

$$r^2 = \left(\frac{a}{h} \cos^2 \theta \left(\frac{a}{h} \cos^2 \varphi + \frac{b}{c} \sin^2 \varphi \right) + \frac{b}{c} \sin^2 \theta \right)$$

$$\Rightarrow V = \iiint_D dx dy dz = \int_0^{2\pi} d\varphi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3 \theta \sin \theta d\theta \int_0^{\frac{1}{\cos^2 \theta (\frac{a}{h} \cos^2 \varphi + \frac{b}{c} \sin^2 \varphi) + \frac{b}{c} \sin^2 \theta}} r^2 dr$$

$$\Rightarrow \frac{4abc}{3} \int_0^{2\pi} \sin^2 \varphi d\varphi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3 \theta \left(\frac{a}{h} \cos^2 \theta \left(\frac{a}{h} \cos^2 \varphi + \frac{b}{c} \sin^2 \varphi \right) + \frac{b}{c} \sin^2 \theta \right)^3 d\theta =$$

$$= \frac{4abc}{3} \int_0^{2\pi} \left(\frac{b}{c} + \cos^2 \varphi \left(\frac{a}{h} - \frac{b}{c} \right) \right)^3 \sin^2 \varphi d\varphi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3 \theta d\theta$$

$$\left. \frac{\cos^4 \theta}{4} \right|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{2}$$

Σ 4316

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = \left(\frac{x}{a}\right)^{n-1} + \left(\frac{y}{b}\right)^{n-1} \quad (a > 0, b > 0, n > 1) \text{ u einem Kreis.}$$

$$y = \frac{b}{a} t, \quad x = \frac{a(1+t^{n+1})}{1+t^n}, \quad y = \frac{bt(1+t^n)}{1+t^n} \quad (0 < b < 2 + \infty)$$

$$x dy - y dx = ab \frac{(1+t^{n+1})}{(1+t^n)^2} dt, \quad S = \frac{ab}{2} \int_0^{\infty} \frac{(1+t^{n+1})^2}{(1+t^n)^2} dt =$$

$$= \frac{ab}{2} \left(\int_0^{\infty} \frac{1+t^{2n+2}}{(1+t^n)^2} dt + 2 \int_0^{\infty} \frac{t^{n+1}}{(1+t^n)^2} dt + \int_0^{\infty} \frac{1}{(1+t^n)^2} dt \right) = \frac{ab}{2} \left(\frac{1}{n} B\left(2-\frac{1}{n}, \frac{1}{n}\right) + \frac{2}{n} B\left(1, 1\right) + \frac{1}{n} B\left(1, 1-\frac{1}{n}\right) \right) = \frac{ab}{n} \left(1 + B\left(2-\frac{1}{n}, \frac{1}{n}\right) \right) = \frac{ab}{n} \left(1 + \left(1-\frac{1}{n}\right) \frac{\Gamma(1-\frac{1}{n}) \Gamma(\frac{1}{n})}{\Gamma(1)} \right) = \frac{ab}{n} \left(1 + \frac{(1-\frac{1}{n}) \Gamma}{\sin \frac{\pi}{n}} \right)$$

7.5.1

$$I = \frac{1}{2\pi} \oint_C \frac{x dy - y dx}{x^2 + y^2}, \quad \begin{matrix} X = ax + by \\ Y = cx + dy \end{matrix} \quad \begin{matrix} (ad - bc) \neq 0 \\ \text{any non-const. } C \end{matrix}$$

$$x dy - y dx = (ax + by)(cdx + ddy) - (cx + dy)(adx + bdy) = (ad - bc)(x dy - y dx)$$

$$I = \frac{1}{2\pi} \oint_C P(x,y) dx + Q(x,y) dy, \quad P = -\frac{(ad - bc)y}{(ax + by)^2 + (cx + dy)^2}, \quad Q = \frac{(ad - bc)x}{(ax + by)^2 + (cx + dy)^2}$$

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} = -\frac{(ad - bc)(a^2 + c^2)x^2 + (b^2 + d^2)y^2}{[(ax + by)^2 + (cx + dy)^2]^2}$$

$$\oint_C P(x,y) dx + Q(x,y) dy = \oint_{C'} P(x,y) dx + Q(x,y) dy \quad \left| \begin{matrix} C': (ax + by)^2 + (cx + dy)^2 = r^2 \\ x^2 + y^2 = r^2 \end{matrix} \right.$$

$$I = \frac{1}{2\pi} \oint_{x^2 + y^2 = r^2} \frac{x dy - y dx}{x^2 + y^2} = \frac{1}{2\pi r^2} \oint_{x^2 + y^2 = r^2} x dy - y dx = \frac{ad - bc}{2\pi r^2} \oint_{x^2 + y^2 = r^2} x dy - y dx =$$

$$= \frac{ad - bc}{2\pi r^2} \iint_{x^2 + y^2 \leq r^2} 2 dx dy = \frac{ad - bc}{\pi r^2} \iint_{x^2 + y^2 \leq r^2} 1 \cdot \frac{P(x,y)}{D(x,y)} dx dy$$

$$\Rightarrow \frac{D(x,y)}{D(x,y)} = ad - bc \Rightarrow \frac{D(x,y)}{D(x,y)} = \frac{1}{ad - bc}, \quad I = \frac{ad - bc}{\pi r^2} \iint_{x^2 + y^2 \leq r^2} \frac{1}{ad - bc} dx dy =$$

$$= \frac{ad - bc}{\pi r^2} \cdot \frac{1}{ad - bc} \cdot \pi r^2 = \text{sign}(ad - bc).$$

Кл. работа

Поверхностные интегралы

вычисл. по формул. и т.д.

Рассмотрим

$$z = \varphi(x, y)$$

век. норм. Т.

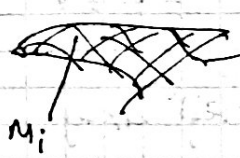
$$\vec{n} = \vec{e}_x \varphi'_x + \vec{e}_y \varphi'_y + \vec{e}_z, \quad (x'_0)^2 + (y'_0)^2 + (z'_0)^2 = 0 \text{ на } \text{пол. Т.}$$

Если элемент $\vec{n}$ и $\vec{r} = (x, y, z) \Rightarrow$ не определено.
 Неправильно по Дираку (??).

Рассм. функции 1-го рода:

$$\iint_S f(x, y, z) dS, \quad z = z(x, y)$$

$$\lim_{n \rightarrow \infty} \sigma \begin{pmatrix} x'_n & y'_n & z'_n \\ x''_n & y''_n & z''_n \end{pmatrix} \Rightarrow \text{не определено Т.}$$



соединяем элемент. $\lim \Sigma n_i$ - угол I погр.

$$\textcircled{2} \iint_D f(x,y,z(x,y)) \sqrt{1+(x'_u)^2+(x'_v)^2} dx dy$$

Результат. 2 пара



$$(P, Q, R) \equiv \vec{A}$$

группа на кол.

$$\iint_S (\vec{A}, \vec{n}) dS$$

$$\vec{n} = \frac{[\vec{e}_u, \vec{e}_v]}{|\vec{e}_u \times \vec{e}_v|}$$

элем-т поверхности

$R(u,v) = (x(u,v), y(u,v), z(u,v))$
или $R(u,v) = (x(u,v), y(u,v), z(u,v))$
или $R(u,v) = (x(u,v), y(u,v), z(u,v))$

$$z = z(u,v) = (x(u,v), y(u,v), z(u,v))$$

$$v = v_0 \quad x(u, v_0), y(u, v_0), z(u, v_0)$$

$$u = u_0 \quad x(u_0, v), y(u_0, v), z(u_0, v)$$



$$z'_u = x'_u, y'_u, z'_u$$

$$z'_v = x'_v, y'_v, z'_v$$



Внутри в одной т. и возмущена элемент. по
вект. кол.

Нормальный
вектор
2-го

4343

$$I = \iint_S (x+y+z) dS, \quad S: x^2+y^2+z^2=a^2, \quad D: x^2+y^2 \leq a^2, z \geq 0.$$

$$z = \sqrt{a^2 - x^2 - y^2}$$

$$z'_x = \frac{-x}{\sqrt{a^2 - x^2 - y^2}}, \quad z'_y = \frac{-y}{\sqrt{a^2 - x^2 - y^2}}$$

$$dS = \sqrt{1+(z'_x)^2+(z'_y)^2} dx dy$$

$$I = \iint_{x^2+y^2 \leq a^2} (x+y+\sqrt{a^2-x^2-y^2}) \cdot \frac{a dx dy}{\sqrt{a^2-x^2-y^2}}$$

$$\int_0^{2\pi} \int_0^a r dr = 2\pi a \cdot \frac{a^2}{2} = \pi a^3.$$

$$\int_0^{2\pi} (\cos \varphi + \sin \varphi) d\varphi = 0.$$

$$= \iint_{x^2+y^2 \leq a^2} \left(\frac{x+y}{\sqrt{a^2-x^2-y^2}} + 1 \right) dx dy = \left| \begin{matrix} x=2a \cos \varphi \\ y=2a \sin \varphi \\ y=2 \\ 0 \leq \varphi \leq 2\pi \\ 0 \leq r \leq a \end{matrix} \right| =$$

4344

$$I = \iint_S (x^2+yz) dS, \quad S: \sqrt{x^2+y^2} \leq z \leq 1$$

$$S = S_1 \cup S_2 \quad dS_1 = \sqrt{1+(z'_x)^2+(z'_y)^2} dx dy$$

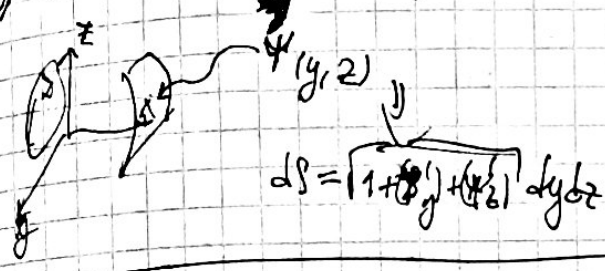


$$z'_x = \frac{x}{\sqrt{x^2+y^2}}, \quad z'_y = \frac{y}{\sqrt{x^2+y^2}}, \quad dS = \sqrt{1 + \frac{x^2+y^2}{x^2+y^2}} dx dy = \sqrt{2} dx dy$$

$$I = I_1 + I_2; \quad I_2 = \iint_{x^2+y^2 \leq 1} (x^2+y^2) \sqrt{2} dx dy + \iint_{x^2+y^2 \leq 1} (x^2+y^2) dx dy = (1+\sqrt{2}) \iint_{x^2+y^2 \leq 1} (x^2+y^2) dx dy =$$

$$= (1+\sqrt{2}) \int_0^{2\pi} \int_0^1 r^2 dr d\varphi = 2\pi(1+\sqrt{2}) \cdot \frac{1}{3} = \frac{\pi(1+\sqrt{2})}{3}$$

Решение $\vec{r} = \vec{r}(u, v) = (x(u, v), y(u, v), z(u, v))$; $dS = \sqrt{1 + E'_x + (z'_y)^2} dx dy$



$$dS = \sqrt{EG - F^2} du dv, \quad E = (r'_u, r'_u), \quad G = (r'_v, r'_v), \quad F = (r'_u, r'_v)$$

Задача (параметризация)

$$I_2 = \iint_S z^2 dS, \quad S: \text{верх конуса}$$

$$x = r \cos \varphi \sin \alpha, \quad y = r \sin \varphi \sin \alpha, \quad z = r \cos \alpha$$

$$0 \leq \alpha \leq \frac{\pi}{2}, \quad 0 \leq \varphi \leq 2\pi$$

$$I = \iint_S \sqrt{EG - F^2} r^2 \cos^2 \alpha dr d\varphi d\alpha$$

$$E = (r'_\alpha, r'_\alpha) = ((\cos \alpha \sin \varphi, \sin \alpha \sin \varphi), (\cos \alpha \cos \varphi, \sin \alpha \cos \varphi)) =$$

$$= 1$$

$$G = (r'_\varphi, r'_\varphi) = (-r \sin \alpha \sin \varphi, r \sin \alpha \cos \varphi, 0) = r^2 \sin^2 \alpha$$

$$F(r_\alpha, r_\varphi) = 0$$

$$\Rightarrow dS = \sqrt{r^2 \sin^2 \alpha} dr d\alpha = r \sin \alpha dr d\alpha; \quad I = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \int_0^1 r^2 \cos^2 \alpha r \sin \alpha dr d\varphi d\alpha =$$

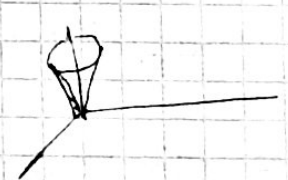
$$= \int_0^{\frac{\pi}{2}} \cos^2 \alpha \sin \alpha d\alpha \int_0^{2\pi} d\varphi \int_0^1 r^3 dr = \frac{1}{2} \cdot 2\pi \cdot \frac{1}{4} = \frac{\pi}{2}$$

Д/П

Задача

$$I = \iint_S |xyz| dS, \quad S: \begin{cases} z = \sqrt{x^2+y^2} \\ x^2+y^2 \leq 1 \\ z \geq 1 \end{cases} \quad \begin{matrix} z'_x = 2x \\ z'_y = 2y \end{matrix} \Rightarrow dS = \sqrt{1 + 4(x^2+y^2)} dx dy$$

$$I = \iint_{x^2+y^2 \leq 1} |xyz| \sqrt{1 + 4(x^2+y^2)} dx dy = \iint_{x^2+y^2 \leq 1} |xyz| \sqrt{1 + 4r^2} r dr d\varphi$$



$$= \int_0^{2\pi} \int_0^1 r^3 \sin \varphi \cos \varphi \sqrt{1 + 4r^2} r dr d\varphi = 2 \int_0^1 r^5 \sqrt{1 + 4r^2} dr =$$

$$= |r^2, t| = 2 \int_0^1 \frac{1}{2} \sqrt{1+4t} dt = \int_0^1 \frac{1}{\sqrt{1+4t}} dt = \frac{1}{2} \left(\frac{y^2}{2} - \frac{2y^5}{5} + \frac{2y^3}{3} \right) \Big|_0^1$$

$y = \sqrt{1+4t}$

$$= \frac{125\sqrt{5} - 1}{420}$$

S 4397

$$\iint_S \frac{dS}{h}, \quad S: \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1; \quad h = \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}}$$

$$\iint_{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1} \frac{c^2 \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \right)}{1 \pm 1} dxdy = 2 \iint_{\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1} \frac{c \left(\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} \right) + \frac{1}{c^2} \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right) \right)}{\sqrt{1 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} \right)}} dxdy =$$

$$= 2 \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \frac{c}{\sqrt{1-r^2}} = \left(\frac{r^2 y^2}{a^2} + \frac{r^2 x^2}{b^2} + \frac{1}{c^2} - \frac{r^2}{c^2} \right) ab \theta d\theta = 2abc \int_0^{\frac{\pi}{2}} \left(\frac{1}{\sqrt{1-r^2}} \left(\frac{1}{a^2} + \frac{1}{b^2} - \sqrt{1-r^2} \cdot \left(\frac{1}{a^2} + \frac{1}{b^2} \right) + 2 \frac{\sqrt{1-r^2}}{c^2} \right) \right) d\theta =$$

$x = ar \cos \theta$
 $y = br \sin \theta$

$$= -abc \left(2\sqrt{1-r^2} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) - \frac{r}{3} (1-r^2)^{\frac{3}{2}} \cdot \left(\frac{1}{a^2} + \frac{1}{b^2} \right) + \frac{4}{3c} (1-r^2)^{\frac{3}{2}} \right) \Big|_0^{\frac{\pi}{2}} =$$

$$= \frac{4\pi}{3} abc \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \right)$$

$$\begin{cases} x = a \sin \theta \cos \varphi \\ y = b \sin \theta \sin \varphi \\ z = c \cos \theta \end{cases} \quad \begin{matrix} 0 \leq \theta \leq \pi \\ 0 \leq \varphi \leq 2\pi \end{matrix}$$

$$I = 8 \iint_{S_1} \frac{dS}{h};$$

$$A = \frac{D(x,y)}{D(\theta,\varphi)}, \quad B = \frac{D(z,y)}{D(\theta,\varphi)}, \quad C = \frac{D(x,y)}{D(\theta,\varphi)}$$

$$\nabla u = C > 0, \quad 0 < \theta < \frac{\pi}{2} \Rightarrow$$

$$bc \sin^2 \theta \cos \varphi$$

$$ac \sin^2 \theta \sin \varphi$$

$$ab \cos \theta \cos \varphi$$

$$\Rightarrow \cos \theta = \frac{A}{\sqrt{A^2 + B^2 + C^2}}$$

$$dS = \sqrt{A^2 + B^2 + C^2} d\theta d\varphi$$

$$I = \iint_S \frac{A^2 + B^2 + C^2}{A x(\theta, \varphi) + B y(\theta, \varphi) + C z(\theta, \varphi)} d\theta d\varphi \quad \textcircled{2}$$

$$\cos \varphi = \frac{B}{\sqrt{A^2 + B^2 + C^2}}$$

$$\textcircled{2} \text{ Subst } \int_0^{\frac{\pi}{2}} \sin \theta d\theta \int_0^{2\pi} \left(\frac{\sin^2 \theta \cos^2 \varphi}{a^2} + \frac{\sin^2 \theta \sin^2 \varphi}{b^2} + \frac{\cos^2 \theta}{c^2} \right) d\varphi$$

$$= 4\pi abc \left(\frac{1}{4} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta + \frac{1}{c^2} \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta \right) = \frac{4}{3} \pi abc \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \right)$$

Kn. pedba

1-ugorno-nazgale, $x = x(u, v)$ ($u, v \in D$)
 $y = y(u, v)$
 $z = z(u, v)$

$$\iint_D f(x(u, v), y(u, v), z(u, v)) \sqrt{EG + F^2} du dv = \int_S f(x, y, z) dS$$

408 $x^2 + y^2 + z^2 = a^2$, $r = a$ $\begin{cases} x = a \cos \varphi \cos \psi \\ y = a \sin \varphi \cos \psi \\ z = a \sin \psi \end{cases}$, $k = \frac{b}{a}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $b \leq a$ $\rightarrow$ brygn ymmygn

$\cos^2 \psi = \frac{k^2}{k^2 \cos^2 \varphi + \sin^2 \varphi}$, $\sin^2 \psi = \frac{(1-k^2) \sin^2 \varphi}{k^2 \cos^2 \varphi + \sin^2 \varphi}$

$\int = \iint d\Omega = 8a^2 \int_0^{\pi/2} d\psi \int_0^{2\pi} \cos \psi d\varphi$ (3)
 $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$
 $x^2 + y^2 + z^2 = a^2$

$b \leq a$, $\int_0^{\pi/2} \frac{\sqrt{1-k^2}}{\sqrt{1-k^2 \cos^2 \varphi}} d\varphi = 8a^2 \left(\frac{\pi}{2} - a \arcsin \sqrt{1-k^2} \right)$

4348

$I = \iint_S z dS$, S - rann nob. rymkowsa. $E = x_u^2 + y_u^2 + z_u^2 = \cos^2 v + \sin^2 v = 1$.
 $\begin{cases} x = u \cos v & 0 \leq u \leq a \\ y = u \sin v & 0 \leq v \leq 2\pi \\ z = v & \end{cases}$ $G = u^2 + 1$
 $F = 0$

$\Rightarrow I = \int_0^a \int_0^{2\pi} \sqrt{1+u^2} du \int_0^{2\pi} v dv = 2\pi^2 \int_0^a \sqrt{1+u^2} du$, $u = \sinh t$; $du = \cosh t dt$

$2\pi^2 \int_0^{\sinh a} \frac{\cosh^2 t}{\cosh 2t + 1} dt = \dots$

9350

$I = \iint_S (xy + yz + zx) dS$ $z = \sqrt{x^2 + y^2}$, $x^2 + y^2 = 2ax$, $(x-a)^2 + y^2 = a^2$

$z'_x = \frac{x}{\sqrt{x^2 + y^2}}$, $z'_y = \frac{y}{\sqrt{x^2 + y^2}}$, $dS = \sqrt{1 + \frac{x^2 + y^2}{x^2 + y^2}} dx dy = \sqrt{2} dx dy$
 $\begin{cases} x = 2a \cos \varphi \\ y = 2a \sin \varphi \end{cases} \varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ $I = \sqrt{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^{2a \cos \varphi} (x^2 \cos \varphi \sin \varphi + y^2 (\cos \varphi + \sin \varphi)) dx =$
 $= \sqrt{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(2a \cos \varphi)^4}{4} (\cos \varphi + \sin \varphi + \cos^2 \varphi + \sin^2 \varphi) d\varphi$

9/p

4041

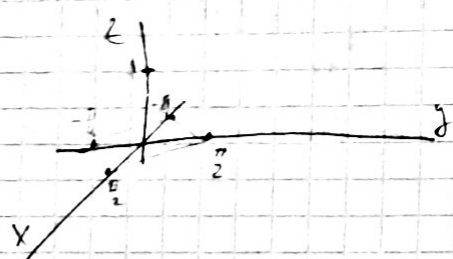
$z = \sqrt{x^2 + y^2}$ поверхность $x^2 + y^2 = 2x$, $(x-1)^2 + y^2 = 1$
 $z'_x = \frac{x}{\sqrt{x^2 + y^2}}$, $z'_y = \frac{y}{\sqrt{x^2 + y^2}}$

$$\int = \iint_{(x-1)^2 + y^2 \leq 1} \sqrt{1 + z'^2_x + z'^2_y} dx dy = \pi \sqrt{2}$$

4048 (6 points)

4040

$z = \cos x \cos y$, $z < 0$ $|x-y| \leq \frac{\pi}{2}$, $|x+y| \leq \frac{\pi}{2}$. $-\frac{\pi}{2} \leq x-y \leq \frac{\pi}{2}$, $-\frac{\pi}{2} \leq x+y \leq \frac{\pi}{2}$



$$V = \int_0^{\arccos \frac{z_0}{\cos x}} S(z) dz$$

$$S(z_0) = 4 \cdot \int_0^{\frac{\pi}{2}} dx \int_0^{\frac{\pi}{2}} dy$$

$z_0 = \cos x \cos y$
 $\Rightarrow \cos y = \frac{z_0}{\cos x} \Rightarrow y = \arccos \frac{z_0}{\cos x}$

$$V = 2 \int_0^{\frac{\pi}{2}} dx \int_{x-\frac{\pi}{2}}^{x-\frac{\pi}{2}} \cos x \cos y dy = 4 \int_0^{\frac{\pi}{2}} \cos^2 x dx = 4 \left(\frac{x}{2} + \frac{\sin 2x}{4} \right) \Big|_0^{\frac{\pi}{2}} = \pi$$

4042



$$x^2 + y^2 + z^2 = R^2$$

$x = R \cos \varphi \sin \psi$
 $y = R \sin \varphi \sin \psi$
 $z = R \cos \psi$

$E = R^2 \sin \psi$
 $G = R^2$
 $F = 0$

$$\int_{\varphi_1}^{\varphi_2} d\varphi \int_{\psi_1}^{\psi_2} R^2 \cos \psi d\psi = (\varphi_2 - \varphi_1) (\sin \psi_2 - \sin \psi_1) R^2$$

Кривая

Итак это S-м. Граничное.



$X = X(u, v)$
 $y = y(u, v)$
 $z = z(u, v)$

Т.е. групп. X, y, z

группы - некие T. и т.д. (какие-то группы)

группы. если 3 группы. 6 членов none норм

Выводы, если есть какие-то

$$\arg \begin{pmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \end{pmatrix} < 2$$

$\vec{A} = (P, Q, R)$ имеет на поверхности $\Rightarrow \iint_S (\vec{A}, \vec{n}) dS$ - поток-а вектора $\vec{A}$ через S

$\vec{n} = \frac{[r_u, r_v]}{|[r_u, r_v]|}$, $dS = |[r_u, r_v]| du dv$

вычисляется
как скалярное
произведение

$= \iint_S A_i \frac{[r_u, r_v]_i}{|[r_u, r_v]|} |[r_u, r_v]| du dv$



$x^2 + y^2 + z^2 = a^2 \Rightarrow z = \pm \sqrt{a^2 - x^2 - y^2}$
 Определим K_1 и K_2 (не ген-то).

4362

опр. век. (касательная)

$I = \iint_S (x dy dz + y dz dx + z dx dy)$

$dy dz = \cos \alpha dS$
 $dz dx = \cos \beta dS$
 $dx dy = \cos \gamma dS$

$\cos \alpha, \cos \beta, \cos \gamma$
 направляющие косинусы

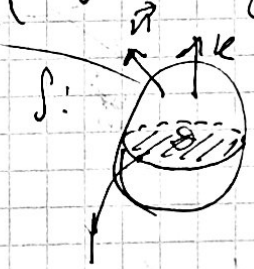
$\gamma: x^2 + y^2 = a^2, z=0$. Уни. бл. по $S^+ - S^-$

$I = \iint_{S^+} z dx dy + \iint_{S^-} z dx dy =$

$= \iint_{S^+} z^+ dx dy - \iint_{S^-} z^- dx dy =$

$= 2 \iint_D \sqrt{a^2 - x^2 - y^2} dx dy =$

$= 2 \iint_{x^2+y^2 \leq a^2} \sqrt{a^2 - x^2 - y^2} dx dy = \int_0^{2\pi} \int_0^a \sqrt{a^2 - r^2} r dr d\varphi =$



$x^2 + y^2 + z^2 = a^2 \Rightarrow z = \pm \sqrt{a^2 - x^2 - y^2}$

$S_+: z = +\sqrt{a^2 - x^2 - y^2}$

$S_-: z = -\sqrt{a^2 - x^2 - y^2}$

$D: x^2 + y^2 \leq a^2, z=0$

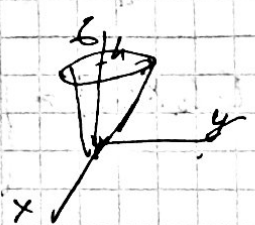
$= \frac{4}{3} \pi a^3$; $I_1 = I_2 = I_3 \Rightarrow 3 \cdot \frac{4}{3} \pi a^3 = 4 \pi a^3 = I$

4364

$I = \iint_S (y-z) dy dz + (z-x) dz dx + (x-y) dx dy$, S : конус

$D: x^2 + y^2 \leq h^2$

$x^2 + y^2 \leq z^2$
 $0 \leq z \leq h$



$I = \iint_S (x-y) dx dy =$

$I = \iint_S (y-z) \cos \alpha + (z-x) \cos \beta + (x-y) \cos \gamma dS \Rightarrow \iint_D (x-y) \cos \gamma dS$

$$z^2 = x^2 + y^2, z = \pm \sqrt{x^2 + y^2}$$

$$z'_y = \frac{x}{\sqrt{x^2 + y^2}}, \cos \varphi = \frac{1}{\sqrt{1 + z_x'^2 + z_y'^2}}$$

$$z'_x = \frac{y}{\sqrt{x^2 + y^2}}$$



$$\cos \beta = \frac{z'_y}{\sqrt{1 + z_x'^2 + z_y'^2}}, \cos \alpha = \frac{z'_x}{\sqrt{1 + z_x'^2 + z_y'^2}}$$

$$ds = \sqrt{1 + z_x'^2 + z_y'^2} dx dy$$

$$I_3 = - \iint_D \frac{(x-y)}{\sqrt{1 + z_x'^2 + z_y'^2}} \sqrt{1 + z_x'^2 + z_y'^2} dx dy = - \iint_D (x-y) dx dy = \int_0^{2\pi} dy \int_0^h z^2 dz = 0$$

Answer: no, det. 2 unit. = 0.

sin φ cos φ

4365

$$I = \iint_S \left(\frac{dy dz}{x} + \frac{dz dx}{y} + \frac{dx dy}{z} \right)$$

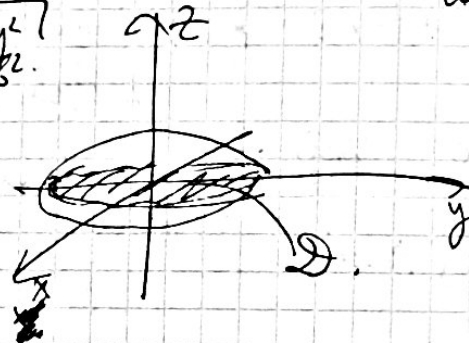
S: lower. cr. sphere.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

→ open. ka blem. kofuom
lower. cr. ydan
cz.

$$\frac{z^2}{c^2} = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \Rightarrow z = \pm c \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$$

~~Handwritten notes~~
x = a cos φ sin θ
y = b sin φ sin θ
z = c cos θ



$$I = \iint_S \left(\frac{\cos \alpha}{x} + \frac{\cos \beta}{y} + \frac{\cos \gamma}{z} \right) dS$$

$$\cos \alpha = \frac{a c}{\sqrt{a^2 b^2 + c^2}}$$

$$A = \begin{vmatrix} y'_\theta & z'_\theta \\ y'_\varphi & z'_\varphi \end{vmatrix} = y'_\theta z'_\varphi - y'_\varphi z'_\theta = b c \sin^2 \theta \sin \varphi$$

$$\cos \beta = \frac{b}{\sqrt{a^2 + b^2 \sin^2 \theta}}$$

$$B = \begin{vmatrix} z'_\theta & x'_\theta \\ z'_\varphi & x'_\varphi \end{vmatrix} = a c \sin^2 \theta \sin \varphi$$

$$\cos \gamma = \frac{a}{\sqrt{a^2 + b^2 \sin^2 \theta}}$$

$$C = \begin{vmatrix} x'_\theta & y'_\theta \\ x'_\varphi & y'_\varphi \end{vmatrix} = a b \sin^2 \theta \cos \varphi > 0$$

$$dS = \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta} d\varphi d\theta$$

$$I = \left(\frac{bc}{a} + \frac{ac}{b} + \frac{ad}{c} \right) \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\varphi = 4\pi abc \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

$$[z_u \times z_v] = \begin{vmatrix} i & j & k \\ x_u & y_u & z_u \\ x_v & y_v & z_v \end{vmatrix} = i|A| + j|B| + k|C|$$

$$A = y_u z_v - y_v z_u$$

$$B = z_u x_v - z_v x_u$$

$$C = x_u y_v - x_v y_u$$

$$\int \sqrt{A^2 + B^2 + C^2} \, dxdy \quad \text{EG-F}$$

D/P

4.363

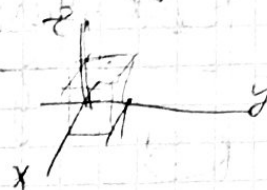
$$\int \int \int f(x,y,z) \, dz \, dy \, dx + g(y) \, dz \, dx + h(z) \, dx \, dy,$$

$$\int \int h(z) \, dx \, dy = \int \int_{\substack{0 \leq x \leq a \\ 0 \leq y \leq b}} h(z) \, dx \, dy =$$

$$f(x), g(y), h(z) \in C;$$

$$S: \text{ку. от. ноб. напар. } -a$$

$$0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c.$$



$$\int \int h(z) \, dx \, dy = abc \cdot \frac{h(c) - h(0)}{c}$$

$$\Rightarrow \int = abc \left(\frac{f(a) - f(0)}{a} + \frac{g(b) - g(0)}{b} + \frac{h(c) - h(0)}{c} \right)$$

4.366

$$\int \int \int x^2 \, dy \, dz + y^2 \, dz \, dx + z^2 \, dx \, dy, \quad S: \text{ку. сфера с р. } (x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$$

$$\int \int z^2 \, dx \, dy = \int \int_{(x-a)^2 + (y-b)^2 \leq R^2} (c + \sqrt{R^2 - (x-a)^2 - (y-b)^2})^2 \, dx \, dy = \int \int_{\substack{z=c \\ z=c+\sqrt{R^2 - (x-a)^2 - (y-b)^2}}} (c + \sqrt{R^2 - (x-a)^2 - (y-b)^2})^2 \, dx \, dy$$

$$= 4\pi \int_0^R \int_0^{2\pi} \int_0^{\pi} (c + \sqrt{R^2 - r^2})^2 \, r \, d\varphi \, dr \, d\theta = 8\pi c \left(-\frac{1}{3} (R^2 - r^2)^{\frac{3}{2}} \right) \Big|_0^R = \frac{8}{3} \pi R^3 c$$

$$\Rightarrow \int = \frac{8}{3} \pi R^3 (a+b+c).$$

4.388

$$g_0; M_0(x_0, y_0, z_0, 1, r). \quad u = \iint \frac{g_0 \, dS}{r}, \quad r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$

$$dS = 2\pi a^2 \sin \varphi \, d\varphi$$

$$r = \sqrt{a^2 + r_0^2 - 2r_0 a \cos \varphi} \quad (0 \leq \varphi \leq \pi)$$

$$u = \iint \frac{g_0 \, dS}{r} = 2\pi a^2 g_0 \int_0^\pi \frac{\sin \varphi \, d\varphi}{\sqrt{a^2 + r_0^2 - 2r_0 a \cos \varphi}}, \quad u^2 = a^2 + r_0^2 - 2r_0 a \cos \varphi, \quad r_0 = N_0(a, 0, 0),$$

$$\sin \varphi \, d\varphi = \frac{u}{r_0 a} \, du \Rightarrow u = \frac{2\pi a g_0}{r_0} \int_{|a-r_0|}^{a+r_0} \frac{du}{u} = \begin{cases} 4\pi a g_0, & r_0 < a \\ \frac{4\pi a^2 g_0}{r_0}, & r_0 > a \\ 4\pi a g_0, & r_0 = a \end{cases} \Rightarrow u = 4\pi g_0 \cdot \min(a, \frac{a^2}{r_0}).$$

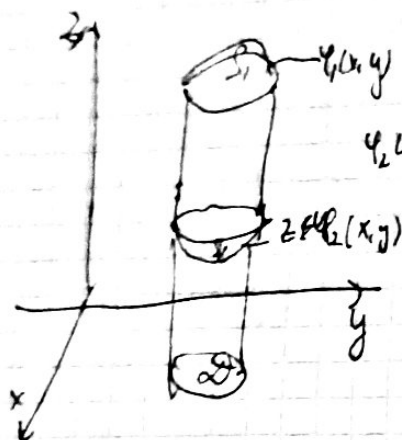
4.397

$$x = r \cos \varphi, \quad y = r \sin \varphi, \quad z = r \quad (0 \leq \varphi \leq 2\pi, 0 \leq r \leq a) \quad g_0; \quad m.$$

$$dS = 2\pi r \, dr = 2\pi r^2 \, dr; \quad dZ = \frac{k m \cdot 2\pi r \, dr \, g_0}{r^2 + z^2} \cdot \frac{z}{\sqrt{r^2 + z^2}} = \frac{k m g_0 \, dr}{r};$$

$$z = \int \frac{dz}{z} = \ln r + i\theta$$

Вопрос
Граничные условия



$\varphi(x, y)$ Г. обр.
 $\varphi_1(x, y) = \varphi_2(x, y)$

$$A = (P, Q, R) \in C^1(G)$$

$$\frac{\partial P}{\partial x}, \frac{\partial Q}{\partial y}, \frac{\partial R}{\partial z} - \text{усл.}$$

$$\textcircled{a} \iiint_G \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right) dx dy dz = \iint_{\partial G} (A, \vec{n}) dS$$

Г
окр.
объем.
масс.

$$\textcircled{b} \iiint_G \operatorname{div} \vec{A} dx dy dz$$

$\iint (P \cos \alpha + Q \cos \beta + R \cos \gamma)$

Ф-ла Грина

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_{\partial D} P dx + Q dy$$

Корн, Кривая
Расс, Тегм
ранд.

5476

$$\iint_S x^2 dy dz + y^2 dx dz + z^2 dx dy = \iint_S (x^2 \cos \alpha + y^2 \cos \beta + z^2 \cos \gamma) dS = \iint_S 3(x^2 + y^2 + z^2) dS$$

$\cos \alpha, \cos \beta, \cos \gamma$ - комп-е ко-
 $\vec{n} = \frac{(x, y, z)}{|(x, y, z)|}$

$$\text{Итак } D: x^2 + y^2 + z^2 \leq a^2 \Rightarrow$$

$$\Rightarrow 3 \iiint_{x^2+y^2+z^2 \leq a^2} (x^2 + y^2 + z^2) dx dy dz; \quad \rho^2 \sin \theta - \text{экв}$$

$$\Rightarrow 3 \int_0^{2\pi} d\varphi \int_0^\pi \sin \theta d\theta \int_0^a r^4 dr = 12\pi \frac{a^5}{5}$$

$$\begin{aligned} x &= r \sin \theta \cos \varphi \\ y &= r \sin \theta \sin \varphi \\ z &= r \cos \theta \end{aligned}$$

5477

$$\iint_S \underbrace{yz}_{\cos \alpha} dy dz + \underbrace{xz}_{\cos \beta} dx dz + \underbrace{xy}_{\cos \gamma} dx dy = \iint_S (P \cos \alpha + Q \cos \beta + R \cos \gamma) dS = 0$$

5478

$$I_2 = \iint_S \frac{x \cos \alpha + y \cos \beta + z \cos \gamma}{\sqrt{x^2 + y^2 + z^2}} dS, \quad P = \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \quad Q = \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \quad R = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$I_2 = \iiint_D (P'_x + Q'_y + R'_z) dx dy dz, \quad P'_x = \frac{x^2 + y^2 + z^2 - x^2}{(x^2 + y^2 + z^2)^{3/2}} = \frac{y^2 + z^2}{(x^2 + y^2 + z^2)^{3/2}}$$

$$\Rightarrow Q'_y = \frac{x^2 + z^2}{(x^2 + y^2 + z^2)^{3/2}}, \quad R'_z = \frac{x^2 + y^2}{(x^2 + y^2 + z^2)^{3/2}}$$

$$I = \iiint_D \frac{2xyz^2 + z^4}{(x^2 + y^2 + z^2)^{3/2}} dx dy dz = \iiint_D \frac{z}{\sqrt{x^2 + y^2 + z^2}} dx dy dz.$$

Угловое. φ :



$$\begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \\ z = z \end{cases}$$

$$J = \begin{vmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1.$$

$$I = \iint_S \left(\frac{\partial x}{\partial \alpha} \cos \alpha + \frac{\partial x}{\partial \beta} \cos \beta + \frac{\partial x}{\partial z} \cos \gamma \right) dS = \iiint_G \underbrace{(u''_{xx} + u''_{yy} + u''_{zz})}_{\Delta u} dx dy dz = \iiint_G \Delta u dx dy dz.$$

(grad $u, \vec{n}$) = $\frac{\partial u}{\partial \vec{n}}$

$$\iint_S \frac{\partial u}{\partial \vec{n}} dS = \iiint_G \Delta u dx dy dz, \quad \text{Ум-е } \vec{n} \text{ - вектор } \vec{n}; \quad \Delta u = 0$$

$$\Rightarrow \iint_S \frac{\partial u}{\partial \vec{n}} dS = 0.$$

$$\iint_S \left[\underbrace{\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right)}_{P_1} \cos \alpha + \underbrace{\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right)}_{Q_1} \cos \beta + \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)}_{R_1} \cos \gamma \right] dS = \iiint_G (P'_x + Q'_y + R'_z) dx dy dz =$$

$$= \iiint_G (R''_{yx} - Q''_{zx} + R''_{zy} - R''_{xy} + Q''_{xz} - P''_{yz}) dx dy dz = 0; \quad A = (P, Q, R)$$

$$\iint_S (\text{rot } A, \vec{n}) dS = \iiint_G \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \cos \alpha + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \cos \beta + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \cos \gamma dS = 0.$$

$$\iint_S (x \cos \alpha + y \cos \beta + z \cos \gamma) dS = \iiint_V dx dy dz = 3|V| \Rightarrow V = \frac{1}{3} \iint_S (x \cos \alpha + y \cos \beta + z \cos \gamma) dS$$

$$\text{Ф-на Грина: } \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_{\partial D} (P \cos \alpha + Q \sin \alpha) dS$$

$$\text{Пусть } P=R, Q=0 \Rightarrow \oint_{\partial D} P dS = \oint_{\partial D} x \sin \alpha dS; \quad |D| = \frac{1}{2} \oint_{\partial D} P dx + Q dy =$$

4382

4383

5-76, 100 V конуса, ор. м. кон. ноб-то



1. Для $g(x) = 0$ - 2n. кон. ноб. и $Ax + By + Cz + D = 0$

$V = \frac{1}{3} \int h$, h - высота конуса

$$\int = \int_{\text{ор}} + \int_{\text{кон}}, \quad (P, \vec{n}) = 0. \quad V = \frac{1}{3} \iint_S (z, \vec{n}) dS + \frac{1}{3} \iint_S (z, \vec{n}) dS =$$

$\int_{\text{ор}} \rightarrow 0. \quad \int_{\text{кон}}$

$$= \frac{1}{3} \iint_{\text{сн.}} (0x + 0y + 1H) dS = \frac{1}{3} HS.$$

Δ4353

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \text{оператор Лапласа}$$

$$\iint_S u \frac{\partial u}{\partial \nu} dS = \iint_S \left(u \left(\frac{\partial u}{\partial x} \cos \alpha + \frac{\partial u}{\partial y} \cos \beta + \frac{\partial u}{\partial z} \cos \gamma \right) \right) dS = \iint_S \left(\frac{\partial}{\partial x} \left(u \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(u \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(u \frac{\partial u}{\partial z} \right) \right) dS$$

$$\iiint_V (\text{grad } u, \text{grad } u) dx dy dz + \iint_S u^2 dx dy dz \quad \ominus$$

$$\ominus \iint_S u \frac{\partial u}{\partial \nu} dS.$$

Δ4394

$$\iiint_V (u \Delta v - v \Delta u) dx dy dz = 0, \quad u, v \in C^2(E), \quad \text{гб. хемп. гур. } \partial V$$

$$\iint_S \left(u \frac{\partial v}{\partial \nu} - v \frac{\partial u}{\partial \nu} \right) dS = \iint_S \left(v \frac{\partial u}{\partial x} \cos \alpha + v \frac{\partial u}{\partial y} \cos \beta + v \frac{\partial u}{\partial z} \cos \gamma - \left(u \frac{\partial v}{\partial x} \cos \alpha + u \frac{\partial v}{\partial y} \cos \beta + u \frac{\partial v}{\partial z} \cos \gamma \right) \right) dS$$

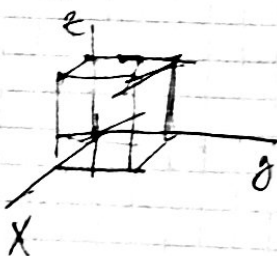
$$= \iiint_V (v \Delta u - u \Delta v) dx dy dz,$$

$$v \frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \left(v \frac{\partial u}{\partial x} \right) - \frac{\partial v}{\partial x} \frac{\partial u}{\partial x}$$

2/p

Δ4377

$$\iint_S x^2 dy dz + y^2 dz dx + z^2 dx dy, \quad S: \text{бл. с. } u \text{ в } \Omega, \quad 0 \leq x \leq a, 0 \leq y \leq a, 0 \leq z \leq a.$$



$$2 \iiint_{\Omega} (x+y+z) dx dy dz = 2 \int_0^a dx \int_0^a dy \int_0^a (x+y+z) dz = 2 \int_0^a dx \int_0^a dy \left(x(y+a) + y \frac{a^2}{2} + \frac{a^3}{2} \right) = 2 \int_0^a dx \left(x \frac{a^2}{2} + y \frac{a^2}{2} + \frac{a^3}{2} \right) = 2 \int_0^a \left(\frac{a^3}{2} + \frac{a^3}{2} + \frac{a^3}{2} \right) dx = 2 \int_0^a \frac{3a^3}{2} dx = 3a^4$$

$$= 2a^4 + a^4 = 3a^4$$

Δ4385

$$\iint_S x^2 dy dz + y^2 dz dx + z^2 dx dy, \quad S - \text{бл. с. } u \text{ в } \Omega, \quad x^2 + y^2 + z^2 = a^2$$

$$4 \cdot \frac{1}{3} \iiint_{\Omega} (x^2 + y^2 + z^2) dx dy dz = 4 \cdot \frac{1}{3} \cdot \frac{4}{3} \pi a^5 = \frac{16}{9} \pi a^5$$

$$\frac{a^4}{2} + \frac{a^4}{2} + \frac{a^4}{2} = \frac{3a^4}{2}$$



$$V = \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} dz \, dy \, dx = \int_0^{2\pi} d\varphi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\psi \int_0^a r^2 \cos \psi \, dr = 6\pi \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \psi \, d\psi \right) \left(\int_0^a r^2 \, dr \right) = \frac{12\pi}{5} a^3$$

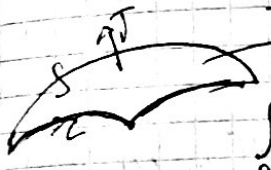
2.1.99

$\int (1-y+z) dy dz + (y-z+x) dz dx + (z-x+y) dx dy \odot$; S - б.в. кр. н.о.б. $|x-y+z| + |y-z+x| + |z-x+y| = 1$

$\odot \int \int \int dx dy dz$, $V: |x-y+z| + |y-z+x| + |z-x+y| = 1$
 $u = x-y+z$
 $v = y-z+x$
 $w = z-x+y$
 $\frac{D(u,v,w)}{D(x,y,z)} = 4$ $|u|+|v|+|w|=1$

$I = \int \int \int_{|u|+|v|+|w| \leq 1} 3 \cdot \frac{1}{4} du dv dw = \frac{3}{4} \cdot \frac{4}{3} = 1$

Кл. работа
Физ. Сурса

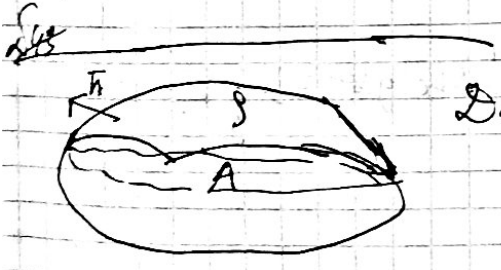


$A(P, Q, R)$ - нек. вып. или согнут-я S , $V \subset \mathbb{R}^3$
 $\int_S (\text{rot } \vec{A}, \vec{n}) dS = \oint_C (A, d\vec{r})$, $d\vec{r}$ - касат. к C .
уравнение $\vec{A}$ по C .

$\text{rot } \vec{A} = [\nabla, \vec{A}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \vec{i} (R_y - Q_z) + \vec{j} (\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}) + \vec{k} (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})$

показ = вып. б. н.о.б.

$\int_S ((R'_y - Q'_z) \cos \alpha + (P'_z - R'_x) \cos \beta + (Q'_x - P'_y) \cos \gamma) dS = \oint_C P dx + Q dy + R dz$



$D \in \mathbb{R}^3$ $\int_S (\vec{A}, \vec{n}) dS = \text{норм. б.в. н.о.б. з.в.з. } S$
 $= \int \int \int \text{div } A \, dx dy dz$ - габит.
 $\odot \text{div } A = (\vec{A}, \nabla) \odot$

$\odot \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$; $\text{rot } A = [\vec{A}, \nabla]$

$\nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}$

rot и div не-век. от CK
 $\int_S (\text{rot}(CK), \vec{n}) dS = \oint CK$
 $d\vec{r} = dx dy dz$

Ум. р.: $\text{div}(CK) = \lim_{\Delta \rightarrow 0} \frac{\int \vec{A} \cdot \vec{n} dS}{|\Delta|}$ Классический
закон Гаусса
и Грина

4445

$$\vec{A} = \bar{i}x^3 + \bar{j}y^3 + \bar{k}z^3, \quad P=x^3, Q=y^3, R=z^3.$$

Найти поток вектора $\vec{A}$ через поверхность $x^2+y^2+z^2=x \Rightarrow$

$$\Rightarrow \iint_S (\vec{A}, \vec{n}) dS = \iiint_V \text{div}(\vec{A}) dx dy dz = \iiint_V (3x^2 + 3y^2 + 3z^2) dx dy dz \quad \text{в } \Omega;$$

$$\varphi^2 = \sin^2 \theta \cos^2 \varphi$$

$$\Omega = \{ (x,y,z) : x^2+y^2+z^2 \leq x \}$$

$$\Rightarrow \int_0^{2\pi} \int_0^{\pi} \int_0^{2\cos\theta} (3x^2 + 3y^2 + 3z^2) r^2 \sin\theta dr d\theta d\varphi$$

$$\Rightarrow 0 \leq \theta \leq \pi$$

$$-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}$$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^{2\cos\theta} (3x^2 + 3y^2 + 3z^2) r^2 \sin\theta dr d\theta d\varphi = \int_0^{2\pi} \int_0^{\pi} (1 - \cos^4 \theta) d\theta d\varphi$$

$$\Rightarrow \int_0^{2\pi} \int_0^{\pi} (1 - \cos^4 \theta) d\theta d\varphi = \int_0^{2\pi} \int_0^{\pi} (1 - \cos^2 \theta) \cos^2 \theta d\theta d\varphi$$

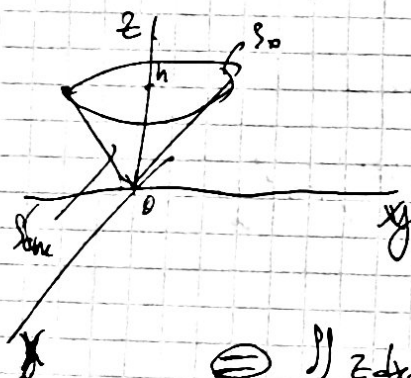
$$= \frac{1}{2} B(1/2, 7/2) = \frac{1}{2} \frac{\Gamma(1/2) \Gamma(7/2)}{\Gamma(4)}$$

$$\Gamma(7/2) = \frac{5 \cdot 3 \cdot 1}{2^3} \Gamma(1/2)$$

$$\frac{150}{2 \cdot 2^5} = \frac{50}{2^5}$$

4442.1

Найти поток вектора $\vec{A} = (x, y, z)$ через конус $K: x^2 + y^2 = z^2, 0 \leq z \leq h$.



$$\text{По гр-е Гаусса: } \iint_S (\vec{A}, \vec{n}) dS = \iiint_V \text{div}(\vec{A}) dx dy dz =$$

$$= \iint_{S_{\text{бок}}} (\vec{A}, \vec{n}) dS + \iint_{S_{\text{осн}}} (\vec{A}, \vec{n}) dS$$

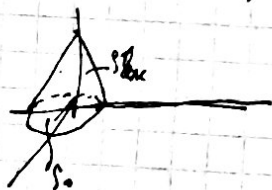
$$= \iint_{S_{\text{бок}}} (z, k) dx dy + \iint_{S_{\text{осн}}} (z, k) dx dy$$

$$\Rightarrow \iint_{S_{\text{бок}}} z dx dy = h \cdot |S_{\text{бок}}| = h \pi h^2$$

$$\Rightarrow h \iint_{S_{\text{бок}}} dx dy = \pi h^3 = \iint_{S_{\text{бок}}} (z, n) dS; \quad \iint_{S_{\text{осн}}} (\vec{A}, \vec{n}) dS = \iint_{K} 3x dy dz = 3 \cdot \frac{1}{3} h S = \pi h^3$$

4443

$\vec{A} = \vec{r}(x, y, z)$, через $S: z = 1 - \sqrt{x^2 + y^2}, 0 \leq z \leq 1$



$$\iint_S (\vec{A}, \vec{n}) dS = \iint_K \text{div}(\vec{r}) dx dy dz = 3|K| = 3\pi h^2$$

$\sqrt{4492}$
 $\vec{r} = \frac{m}{r^3} \vec{r} = \frac{m}{(x^2+y^2+z^2)^{3/2}} (x, y, z); \iint_S (u, n) dS = m \iint_S \left(\frac{x}{r^3} \cos \alpha + \frac{y}{r^3} \cos \beta + \frac{z}{r^3} \cos \gamma \right) dS =$
 $= m \iint_S \frac{(\vec{r}, \vec{n})}{r^3} dS = m \iint_S \frac{\cos(\vec{r}, \vec{n})}{r^2} dS = m \cdot 4\pi$
 $\textcircled{2} \iint_S x^2 y dz = \begin{cases} x = \rho \cos \varphi \sin \theta \\ y = \rho \sin \varphi \sin \theta \\ z = \rho \cos \theta \end{cases} \quad 0.$
 $\iint_S \left[\left(\frac{x}{r^3} \right)'_x + \left(\frac{y}{r^3} \right)'_y + \left(\frac{z}{r^3} \right)'_z \right] dxdydz =$
 $\sigma = \sqrt{x^2+y^2+z^2}.$

$\text{Реш. } \vec{A} = \vec{i}y + \vec{j}x^2 + \vec{k}x, \text{ область кон. контура поверхности. Этого к.п.}$
 $x^2+y^2 \leq 4, z=3.$
 $\iint_S (\text{rot } \vec{A}, \vec{n}) dS = \oint_C \vec{A} d\vec{r}, \vec{n} - \text{норм. вект. конт. } C.$
 $\oint_C (\vec{A}, d\vec{r}) = \oint_C (Pdx + Qdy + Rdz)$
 $\text{rot } \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x^2 & x \end{vmatrix} = \vec{i} \left(\frac{\partial}{\partial y} x - \frac{\partial}{\partial z} x^2 \right) + \vec{k} (x - 1) = \{0, 2x-1\}.$
 $\iint_{x^2+y^2 \leq 4, z=3} (2x-1) dxdy = \int_0^{2\pi} d\varphi \int_0^2 (2r \cos \varphi - 1) r dr = \int_0^{2\pi} \cos \varphi d\varphi \cdot \frac{2r^3}{3} \Big|_0^2 - 2\pi \cdot \frac{r^2}{2} \Big|_0^2 = 4\pi.$
 $\text{Ответ: } 4\pi.$

Diff

$\sqrt{4367}$
 $\vec{I} = \oint_C y dx + z dy + x dz, C: x^2+y^2+z^2=a^2, x^2+y^2+z^2=0; \rightarrow \text{окр.} \rightarrow \text{плоск.} \rightarrow \text{окр.}$
 $I = \iint_S (A \cos \alpha + B \cos \beta + C \cos \gamma) dS = - \iint_S (\cos \alpha + \cos \beta + \cos \gamma) dS = -a^2 (\cos \alpha + \cos \beta + \cos \gamma),$
 $= -\sqrt{3} \pi a^2.$

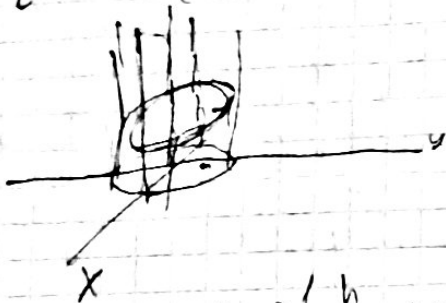
$\sqrt{428}$
 $\oint_{\text{AMB}} (x^2-yz) dx + (y^2-xz) dy + (z^2-xy) dz, \begin{cases} x = a \cos \varphi \\ y = a \sin \varphi \\ z = \frac{h}{2\pi} \varphi \end{cases} \text{ от } A(a, 0, 0) \text{ до } B(a, 0, h)$
 $\int_{\text{AMB}} (x^2-yz) dx + (y^2-xz) dy + (z^2-xy) dz = 0 \Rightarrow$
 $\int_0^h a^2 dz + 0 dx dy + 0 dx dy = 0 \Rightarrow$
 $\Rightarrow \oint_{\text{AMB}} (x^2-yz) dx + (y^2-xz) dy + (z^2-xy) dz = \int_0^h z^2 dz = \frac{h^3}{3}.$

Δ4471

$$\oint_C (y-z)dx + (z-x)dy + (x-y)dz, \quad C: x^2+y^2=a^2, \quad \frac{x}{a} + \frac{y}{b} = 1 \quad (a>0, b>0),$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad y = 1 - \frac{b}{a}x$$

проект. век. $\vec{n} = \vec{i} + \vec{j}$



$$-2 \iint_S dydz + dzdx + dxdy = -2(\cos\alpha + \cos\beta + \cos\gamma) \iint_S dS$$

$$= -2\left(\frac{b}{\sqrt{a^2+b^2}} + 0 + \frac{a}{\sqrt{a^2+b^2}}\right)(\pi a \sqrt{a^2+b^2}) = -2ab(a+b)$$

Аналог.

$$\vec{n} = \frac{\vec{i} + \vec{j}}{\sqrt{2}}$$

$$\frac{1}{\frac{1}{a^2} + \frac{1}{b^2}} = \frac{a^2 b^2}{a^2 + b^2}$$

Кл. работа

$$\vec{A} = (P, Q, R), \quad \oint_C Pdx + Qdy + Rdz = \iint_S (\text{rot } \vec{A}, \vec{n}) dS$$

C — орир. нр. — орир. $\xrightarrow{\text{заключ. от точки } A \text{ и } B}$ $(\Rightarrow) \text{rot } \vec{A} = 0$ $(\Rightarrow) \exists u = u(x, y, z): \text{grad } u = (P, Q, R)$
 none потенциальное $(\Rightarrow)$

На плоскости: $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \Rightarrow \exists u(x, y)$ потенциальн, $\nabla u = \vec{A} = (P, Q); \frac{\partial u}{\partial x} = P, \frac{\partial u}{\partial y} = Q.$

None замкн. выпукл. // неск. и замкн. вып. не замкн. none.

Δ4457

$$\vec{A} = (yz(2x+y+z), xz(y^2+2y+z), xy(x+y+2z))$$

$$\text{rot } \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \vec{i}(x^2+2xy+2xz - x^2-2xy-2xz) + \vec{j}(0) + \vec{k}(0) = 0.$$

$$du = \vec{A} d\vec{r} = Pdx + Qdy + Rdz \Rightarrow xyz(dx+dy+dz) + (x+y+z)(yzdx + xzdy + xydz) = d(xyze)(xyz+e) \Rightarrow u = (xyz+e)(xyz+e) + C = \text{потенциал векторного none.}$$

Δ4458

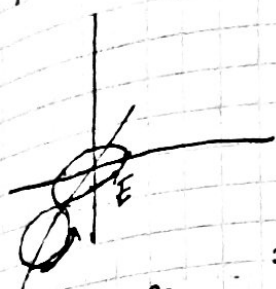
Найти потен. функ. none; $\vec{A} = \frac{m}{r^2} \cdot \vec{r}; \text{rot } \vec{A} = \frac{m}{r^2}(\vec{0} + \vec{j} \times \vec{k}) = 0.$

$$\Rightarrow u = \frac{m}{2r} \cdot \frac{1}{2} \cdot r^2 + C = \frac{1}{2} \frac{m}{r} + C, \quad r = \sqrt{x^2+y^2+z^2}$$

4454
 $A = (1-y, x, z)$, curve?

By using $C_1: x^2 + y^2 = 1$
 $C_2: (x-1)^2 + y^2 = 1$

$\vec{z} = x\vec{i} + y\vec{j} + z\vec{k}$



$\begin{cases} x = \cos t \\ y = \sin t \\ z = 0 \end{cases}, 0 \leq t \leq 2\pi$

$\oint_{C_1} (A, \vec{z}) d\ell = \oint_{C_1} -xy dx + xz dy + cz dz =$

$= \int_0^{2\pi} -\cos t \sin t d(\cos t) + \cos t \sin t d(\sin t) = \int_0^{2\pi} \cos t \sin^2 t + \cos^2 t \sin t dt =$

$\oint_{C_1} (A, d\vec{r}) = \int_0^{2\pi} -\sin t d(\cos t) + \cos t d(\sin t) + c d(c) = 2\pi$

$\oint_{C_2} (-\sin t) d(\cos t + c) + (\cos t + c) d(\sin t) + c d(c) = \int_0^{2\pi} (\sin^2 t + \cos^2 t + 1) dt = 2\pi$

4455

$\text{div}(u \text{grad} u) = (\nabla, u \text{grad} u) = (\nabla u, \text{grad} u) + u(\nabla, \text{grad} u) = (\nabla u, \nabla u) + u \Delta u$

$\nabla = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k}$

$\Delta u = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$

$\text{div}(u \text{grad} v) = (\nabla, u \text{grad} v) = (\nabla u, \text{grad} v) + u(\nabla, \text{grad} v) = (\nabla u, \nabla v) +$

$+ u(\nabla, \nabla v) = \frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial y} + \frac{\partial u}{\partial z} \cdot \frac{\partial v}{\partial z} + u \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$

4455

$\text{rot}(u \vec{R}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ uR_x & uR_y & uR_z \end{vmatrix} = \vec{i} \left(\frac{\partial(uR_z)}{\partial y} - \frac{\partial(uR_y)}{\partial z} \right) + \vec{j} \left(\frac{\partial(uR_x)}{\partial z} - \frac{\partial(uR_z)}{\partial x} \right) + \vec{k} \left(\frac{\partial(uR_y)}{\partial x} - \frac{\partial(uR_x)}{\partial y} \right)$

$[\nabla, u \vec{R}] = [\nabla u, \vec{R}] + u [\nabla, \vec{R}] = u \text{rot}(\vec{R}) + [\text{grad} u, \vec{R}]$

$\vec{F} = k \cdot \vec{z}$, $\text{div}(\vec{F}) = (\nabla, \vec{F}) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$

$\frac{\partial}{\partial x} \left(\frac{k}{2} (x^2 + y^2 + z^2) \right) = \frac{k}{2} (2x) = kx$
 $\Rightarrow \sum \frac{\partial F_i}{\partial x_i} = 0$

Роботинский оператор
Роботинский оператор

Вектор $\vec{A}$ не перпендикулярен.

grad	grad	div	rot
grad	$\times$	grad div α	$\times$
div	div grad $u = \Delta u$	$\times$	div rot $\alpha = 0$
rot	rot grad $u = 0$	$\times$	rot rot $\alpha = \text{grad div } \alpha - \Delta \alpha$

$$\text{div rot } \vec{a} = (\nabla, \text{rot } \vec{a}) = (\nabla, [\nabla, \vec{a}]) = 0.$$

$$\text{rot grad } u = [\nabla, \text{grad } u] = [\nabla, (\nabla, u)] = 0.$$

$$\text{rot}(\text{rot } \vec{a}) = [\nabla, \text{rot } \vec{a}] = [\nabla, [\nabla, \vec{a}]] = \nabla(\nabla, \vec{a}) - \vec{a}(\nabla, \nabla) = \text{grad div } \vec{a} - \Delta \vec{a}.$$

2/P

4.30 (Kraut)

4.39

$$\begin{aligned} \text{rot}(\text{grad } u) &= 0 \\ \text{div}(\text{rot } \vec{a}) &= 0 \end{aligned} \quad \left| \begin{aligned} \text{rot}(\vec{A}) &= [\nabla, \vec{A}] \\ \text{div}(\vec{A}) &= (\nabla, \vec{A}) \end{aligned} \right.$$

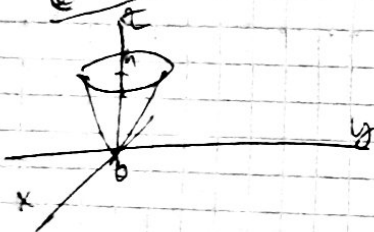
$$1) [\nabla, \text{grad } u] = [\nabla, \frac{\partial u}{\partial x^i} \vec{e}_i] = \frac{\partial^2 u}{\partial x^i \partial x^j} \vec{e}_j - \frac{\partial^2 u}{\partial x^j \partial x^i} \vec{e}_i = 0$$

$$2) \text{div}(\text{rot } \vec{a}) = (\text{rot } \vec{a}, \nabla) = ([\nabla, \vec{a}], \nabla) = 0.$$

4.41 (a)

1) $\vec{r}$ - vektor. $\vec{r} = (x, y, z)$ von $x^2 + y^2 \leq z^2$ ($0 \leq z \leq h$)
2) $\vec{r}$ - vektor. $\vec{r} = (x, y, z)$ von $x^2 + y^2 \leq z^2$ ($0 \leq z \leq h$)

$$\iint_S (\vec{r}, \vec{n}) dS = \iint_S \text{div } \vec{r} dxdydz = \iint_S (x + y + z) dxdydz = \iint_S (x + y + z) dxdydz$$



$$\iint_S (x \cos \alpha + y \cos \beta + z \cos \gamma) dS$$

$$\int_0^h \int_0^{2\pi} \int_0^{\sqrt{z^2 - y^2}} (x + y + z) dx dy dz$$

4.42 (a)

$$\vec{a} = (yz, xz, xy)$$



$$\text{div } \vec{a} = y + x + y = 2y$$

$$\iint_S (\vec{a}, \vec{n}) dS = \iint_S x y dxdy = \int_0^{2\pi} \int_0^a r^2 \sin \varphi dr d\varphi = \int_0^{2\pi} \frac{1}{2} a^3 \sin \varphi d\varphi = 0$$



$$(\vec{r}, \vec{n}) = (\vec{r}, \vec{r}) = r^2$$

$$= \frac{1}{24} \int_0^{2\pi} \sin 2\varphi d2\varphi \int_0^a r^3 dr = \frac{1}{4} \underbrace{(-\cos 2\varphi) \Big|_0^{2\pi}}_{-1+1} \cdot \frac{a^4}{4} = 0 \Rightarrow 0.$$

4452 (1)

$$f(x, y, z); \vec{E}(a \cos t, a \sin t, bt) \quad (0 \leq t \leq 4\pi)$$

$$A = \int_S F ds = \int_C \vec{a} dt \Leftrightarrow$$

$$\Leftrightarrow \int_0^{4\pi} (-a^2 \sin t + a^2 \cos t + b^2 t) dt = b^2 \frac{4\pi^2}{2} = 2\pi^2 b^2.$$

4459

$$a(x, y, z); \quad \begin{cases} a) x^2 + y^2 = 1 \\ b) (x-2)^2 + y^2 = 1, z=0 \end{cases}$$

$$\oint_C (\vec{A}, \vec{r}) d\vec{r} = \iint_S (\text{rot } \vec{A}) d\vec{S}$$

4455

$$a\left(\frac{y}{x}, -\frac{x}{y}, \sqrt{xyz}\right)$$

$$\text{rot } a = \vec{c} + M(x, y)$$

$$P = \text{с. криво}$$

$$\begin{cases} (x-1)^2 + (y-1)^2 + (z-1)^2 = \epsilon^2 \\ (x-1)(\cos \alpha + (y-1) \cos \beta + \\ + (z-1) \cos \gamma) = 0 \\ \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \end{cases}$$

$$\text{rot } a = [\nabla, a] = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{y}{x} & -\frac{x}{y} & \sqrt{xyz} \end{vmatrix} \Leftrightarrow$$

$$\Leftrightarrow \left(\frac{1}{2} \sqrt{\frac{xz}{y}} - \frac{1}{2} \frac{x}{z}, -\frac{1}{2} \sqrt{\frac{yz}{x}}, -\frac{1}{2} - \frac{1}{x} \right) \in T.M(x, y, z): (0, -\frac{1}{2}, -1)$$

$$\oint_C (\vec{a}, \vec{r}) d\vec{r} = \iint_S (\text{rot } a, \vec{r}) d\vec{S} = \iint_S \left(\cos \alpha \left(\frac{1}{2} \sqrt{\frac{xz}{y}} - \frac{1}{2} \frac{x}{z} \right) + \cos \beta \left(-\frac{1}{2} \sqrt{\frac{yz}{x}} \right) + \cos \gamma \left(-\frac{1}{2} - \frac{1}{x} \right) \right) d\vec{S}$$

$$z = 1 - \frac{1}{\cos \gamma} ((x-1) \cos \alpha + (y-1) \cos \beta), \quad z'_x = -\frac{\cos \alpha}{\cos \gamma}, \quad z'_y = -\frac{\cos \beta}{\cos \gamma}, \quad \vec{n} = (\cos \alpha, \cos \beta, \cos \gamma)$$

$$(M, \text{rot } L(M)) = -\frac{1}{2} \cos \beta - \cos \gamma \Rightarrow P = - \iint_S (\cos \beta + \cos \gamma) d\vec{S} = - \left(\frac{1}{2} \cos \beta - \cos \gamma \right) \pi \epsilon^2.$$

26

Задачи

B-1

$$\sqrt{1} \sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right), \quad S_n = \frac{1}{2} \left(1 - \frac{1}{1} + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \dots + \frac{1}{n} - \frac{1}{n+1} \right) \\ = \frac{1}{2} \left(1 + \frac{1}{2} + \dots + \frac{1}{n} \right) \xrightarrow{n \rightarrow \infty} \frac{3}{4}.$$

$$\sqrt{2} \sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+3)}} \text{ не сход.} \quad \lim_{k \rightarrow \infty} \frac{\frac{1}{\sqrt{k(k+3)}}}{\frac{1}{k}} = 1 \Rightarrow \text{расх.} \quad \left| \begin{array}{l} \lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \text{const} \\ \Rightarrow \text{сход.} \end{array} \right.$$

$$\sum_{k=1}^{\infty} \left(\frac{2k-1}{2k+3} \right)^k \text{ un. na ch. no y. korm } \lim_{k \rightarrow \infty} \left| \frac{2k-1}{2k+3} \right| = 1 < 2 \text{ or } \text{paek}$$

$$\sum_{k=1}^{\infty} (-1)^k \left(\frac{1 \cdot 3 \cdot (2k-1)}{2 \cdot 4 \cdot 2k} \right)^p \quad \text{no Ser.: } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left(\frac{1 \cdot 3 \cdot (2k+1)}{2 \cdot 4 \cdot (2k+2)} \right)^p \cdot \frac{2 \cdot 4 \cdot 2k}{1 \cdot 3 \cdot (2k-1)} = \lim_{n \rightarrow \infty} \left(\frac{2k+1}{2k+2} \right)^p = \lim_{k \rightarrow \infty} \left(1 - \frac{1}{2k+2} \right)^p = 1$$

no y. paek:

$$\left(1 - \frac{1}{2k+2} \right)^p = 1 - \frac{p}{2k+2} + \frac{p(p-1)}{(2k+2)^2} + o\left(\frac{1}{k}\right)$$

= ①;

$$a_n = 1 - \frac{1}{k} + \frac{b_k}{k^{1+\epsilon}}, \quad \epsilon > 0$$

$\mu = 1 \rightarrow d > 1$ ch.
 $\lambda \leq 1$ paek.

$$O\left(p \frac{1}{k}\right)$$

no y. paek

$$\left(\frac{2k}{2k+2} \right)^k \left(1 - 1 + \frac{p}{2k+2} \right) = \frac{p}{2} > 1$$

$\frac{p}{2} < 1$ paek

$$p=2: \sum_{k=1}^{\infty} (-1)^k \left(\frac{1 \cdot 3 \cdot (2k-1)}{2 \cdot 4 \cdot 2k} \right)^2$$

1.5

$$\sum_{k=1}^{\infty} k x^k \text{ poryadok ch. } = 1$$

$$\frac{d}{dx} \sum_{k=1}^{\infty} x^k = \sum_{k=1}^{\infty} k x^{k-1}, \quad 0 \leq x < 1 \text{ ch. } \int_0^x \sum_{k=1}^{\infty} k t^{k-1} dt = \sum_{k=1}^{\infty} \int_0^x k t^{k-1} dt = \sum_{k=1}^{\infty} t^k \Big|_0^x = \sum_{k=1}^{\infty} x^k = \frac{x}{1-x} - 1 = \frac{x}{1-x}$$

1.5. Uka na p.c.:

$$\sum_{k=1}^{\infty} \frac{x}{[(k-1)x+1][(k+1)]} \quad x \in (0, 2)$$

$$(*) \frac{p_{k+1}}{p_k} = \frac{(2k+1)!!(2k)!!}{(2k-1)!!(2k+1)!!}$$

$$= \left(1 - \frac{1}{2k+2} \right)^2$$

$$1 - \frac{1}{2k+2} + \frac{1}{(2k+2)^2} = 1 - \frac{1}{2k+2}$$

1.5. 2. Sum

$u_k(x) = \text{un. } 1 \text{ Ser. } u_k(x) \rightarrow \text{Ser. } u_k(x) \rightarrow \text{Ser. } u_k(x)$

$$\sum_{k=1}^n \frac{x}{[(k-1)x+1][(k+1)]} = x \left(\frac{1}{x+1} - \frac{1}{kx+1} \right)$$

$$= \frac{(kx+1) - (k-1)x+1}{[(k-1)x+1](kx+1)} = \frac{1}{(k-1)x+1} - \frac{1}{kx+1}$$

$$S_n = \frac{1}{1} - \frac{1}{n+1} + \frac{1}{n+1} - \frac{1}{n+2} + \dots + \frac{1}{(n-1)+1} - \frac{1}{n+1} = 1 - \frac{1}{n+1} \rightarrow 1$$

1. $x \in \text{yordun. } \in \infty$ $\rightarrow$ ∞ $\rightarrow$ ∞ $\rightarrow$ ∞

Пр. graph X и $\text{graph} \cdot \infty \Rightarrow$ $\text{graph} \cdot \infty$ $\Rightarrow$ $\text{graph} \cdot \infty$

$$\Gamma(p) = \int_0^\infty x^{p-1} e^{-x} dx, \quad 0 < p < +\infty$$

$$\Gamma(p+1) = p \Gamma(p) \quad | \text{ learn } p \in \mathbb{N} \rightarrow \Gamma(p) = (p-1)!$$

$$\Gamma(n+\frac{1}{2}) = \frac{(2n-1)!!}{2^n} \Gamma(\frac{1}{2}) = \sqrt{\pi}$$

$$| \text{ learn } 0 < p < 1 \rightarrow \Gamma(p) \Gamma(1-p) = \frac{\pi}{\sin \pi p}$$

$$B(p, q) = \int_0^1 x^{p-1} (1-x)^{q-1} dx, \quad p > 0, q > 0. \quad \Gamma(n+\frac{1}{2}) = \frac{(2n-1)!!}{2^n} \sqrt{\pi}$$

$$B(p, q) = \int_0^\infty \frac{z^{p-1}}{(1+z)^{p+q}} dz$$

$$B(p, q) = \frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)}$$

$0 \leq \varphi \leq 2\pi, \quad \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$
 $x = a \cos \varphi \cos \theta$
 $y = a \sin \varphi \cos \theta$
 $z = a \sin \theta$
 $dx dy dz = a^3 \sin \theta d\varphi d\theta$

$$\frac{\partial D \mathcal{L}}{\partial \mathcal{L}} = a^3 \sin \theta \cos^2 \varphi \cdot \frac{1}{\sin^2 \varphi}$$

$$S = \iint_D \sqrt{EG-F^2} du dv$$

$$\oint_C P dx + Q dy + R dz = \iiint_V \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} dV$$

$$\sqrt{x^2+y^2} dt = \sqrt{r^2+r'^2} d\varphi$$

$$|D| = \frac{1}{2} \int_C x dy - y dx$$

$$\oint_C (\bar{A}, \bar{E}) d\mathcal{L} = \iint_S \text{rot}(\bar{A}), \bar{n} d\mathcal{S}$$

$$\frac{1}{n-1} > \frac{1}{n} - \text{период.}$$

$$\forall \varepsilon > 0 \exists N(\varepsilon): \forall n \geq N \quad \left| \frac{1}{n-1} - \frac{1}{n} \right| < \varepsilon$$

пр. Абеля $\sum u_n v_n$

$\sum u_n(x)$ с.р. п.р.н.

$|v_n| \leq C \quad \forall x \in E_k$

$v_n(x) \rightarrow 0$ монот. при $k > k_0$.

пр. Дирхле $\sum u_n v_n$

$$\left| \sum_{k=1}^n u_k(x) \right| \leq C$$

$v_n \geq 0$ монотонно при $n \rightarrow \infty$

пр. Абеля-Дирхле $\sum u_n v_n$

$\sum_{k=1}^n u_k(x)$ п.р.н. с.р. u_n монот. с.р. v_n монот.

$$\left| x^n - x^{n-1} - 1 \right| = P_n$$

$$(-1)^n \leq x^n \leq (1)^n$$

$$(-1)^{n-1} \leq x^{n-1} \leq (1)^{n-1}$$

$$\sum_{k=1}^n x^{k-1} (x-1)$$

$$(-1)^n + (-1)^{n-1} \leq x^n - x^{n-1} \leq 0$$

$$S_n = -1 + x^n - 1$$

$$n \log x = \dots$$

$$\sin x \cos y =$$

$$= \frac{1}{2} (2 \sin x \cos y)$$

$$\log \frac{x+y}{x-y} = \log \frac{x+y}{x-y}$$

$$u_n \dots$$

$$\sum_{k=1}^{\infty} \frac{\sin \frac{\pi k}{3}}{k^2}$$

Уч. на сн.

$$\sum_{k=1}^{\infty} \frac{\sin \frac{\pi k}{3}}{k^2} \frac{1}{\left(1 + \frac{\sin \frac{\pi k}{3}}{k^2}\right)^2}$$

$$\sum_{k=1}^{\infty} \left(0 - \frac{\sin \frac{\pi k}{3}}{k^2}\right) \frac{1}{k^2}$$

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Р. Кош, Гаусс, Эйлер

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3923, 3901, 3902

Письм-сговинный лист; y'ye

Григорьев; y'ye

Александров, Лавров, Чубриков;

Смир.

$$\ln(1+x) = \frac{x}{1} - \frac{x^2}{2!} + \frac{x^3}{3!} - \dots + \frac{(-1)^{n-1} x^n}{n!} + \dots$$

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + \frac{(-1)^{n-1} x^{2n}}{(2n)!} + \dots$$

$$\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots$$

Курьевел, курь мат. анализа.

Письм. Словес. Дает. курь работ. см., р. Вейсман. Адамс Дев. Адамс II

Эндрю, Оуэрен Контрпримеры

Курь-курь. работ. см. резул

теор. о курь. курь работ. см. резул

Курс анализа.

Григорьев Письма, Словес.

Оуэрен мат. анализ Вейсман

$$\frac{n(p-x)}{p+x} > 1$$

$$477-453 \Phi$$

$$p-x$$

$$np + nd \leq p + n$$

$$n(p-x+1) \leq p$$

$$n \geq \frac{p}{p-x+1}$$

Характер. Курь работ.

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$$\frac{12}{(n+1)^2} \cdot \frac{1}{(n+1)^2} \ln(n+1)^3$$

$$x = -x;$$

$$\begin{array}{r} x^2 + 5x - 6 \quad | \quad x-3 \\ -x^2 - 3x \\ \hline 8x - 6 \end{array}$$

$$2 = 2.5 + 2.4 = 4.9 \quad x_{11} = \frac{-5 \pm 2}{2} = -6; 1$$

$$\frac{1}{(n+1)(n+2)(n+3)} = -\frac{1}{2} \cdot \frac{1}{n} + \frac{1}{2} \left(-\frac{1}{(n+1)} + \frac{1}{(n+2)} - \frac{1}{(n+3)} \right)$$

Тр. инт. Инновацион. Рухомон
Кур, Мат. анализ, 3 тап

Иннов. 6 об. инт. | Двойн. кинемат. индексы

Матрица инт.

Дополн. индексы
Кинемат. инт.
Матрица инт.

Кинемат. инт.
Кур, Мат. анализ, 3 тап

Двойн. инт.
Кинемат. инт.
Матрица инт.

$$h = x \cos \alpha + y \cos \beta + z \cos \gamma$$

Кур-ла инт.

Кинемат. инт.

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3930, 3935, 3936
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4077, 4083, 4085
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